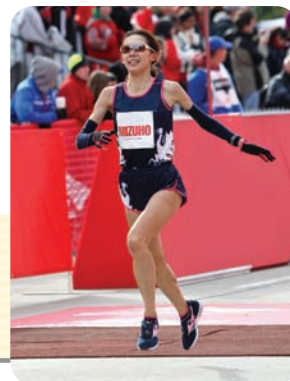# 4.3 Write Linear Equations in Point-Slope Form

Before	You wrote linear equations in slope-intercept form.
Now	You will write linear equations in point-slope form.
Why?	So you can model sports statistics, as in Ex. 43.

Consider the line that passes through the point (2, 3) with a slope of $\frac{1}{2}$.

Let (x, y) where $x \neq 2$ be another point on the line. You can write an equation relating x and y using the slope formula, with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (x, y)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Write slope formula.}$$

$$\frac{1}{2} = \frac{y - 3}{x - 2} \qquad \text{Substitute } \tfrac{1}{2} \text{ for } m, 3 \text{ for } y_1, \text{ and } 2 \text{ for } x_1.$$

$$\frac{1}{2}(x - 2) = y - 3 \qquad \text{Multiply each side by } (x - 2).$$

USE POINT-SLOPE FORM
When an equation is in point-slope form, you can read the x- and y-coordinates of a point on the line and the slope of the line.
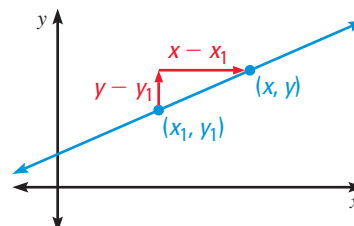
The equation in *point-slope form* is $y - 3 = \frac{1}{2}(x - 2)$.

KEY CONCEPT — *For Your Notebook*

Point-Slope Form

The **point-slope form** of the equation of the nonvertical line through a given point (x_1, y_1) with a slope of m is $y - y_1 = m(x - x_1)$.

EXAMPLE 1 **Write an equation in point-slope form**

Write an equation in point-slope form of the line that passes through the point $(4, -3)$ and has a slope of 2.

$$y - y_1 = m(x - x_1) \qquad \text{Write point-slope form.}$$

$$y + 3 = 2(x - 4) \qquad \text{Substitute 2 for } m, 4 \text{ for } x_1, \text{ and } -3 \text{ for } y_1.$$

✓ **GUIDED PRACTICE** for Example 1

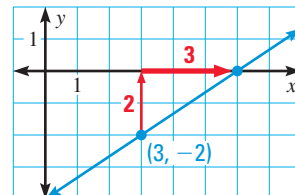1. Write an equation in point-slope form of the line that passes through the point $(-1, 4)$ and has a slope of -2.

EXAMPLE 2   **Graph an equation in point-slope form**

Graph the equation $y + 2 = \frac{2}{3}(x - 3)$**.**

Solution

Because the equation is in point-slope form, you know that the line has a slope of $\frac{2}{3}$ and passes through the point $(3, -2)$.

Plot the point $(3, -2)$. Find a second point on the line using the slope. Draw a line through both points.

✓ **GUIDED PRACTICE**   **for Example 2**

2. Graph the equation $y - 1 = -(x - 2)$.

EXAMPLE 3   **Use point-slope form to write an equation**

Write an equation in point-slope form of the line shown.

Solution

STEP 1 **Find** the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{-1 - 1} = \frac{2}{-2} = -1$$

STEP 2 **Write** the equation in point-slope form. You can use either given point.

Method 1 Use $(-1, 3)$.

$y - y_1 = m(x - x_1)$

$y - 3 = -(x + 1)$

Method 2 Use $(1, 1)$.

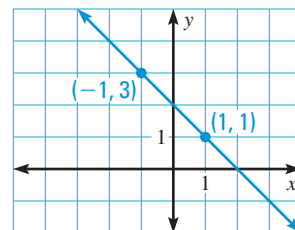$y - y_1 = m(x - x_1)$

$y - 1 = -(x - 1)$

CHECK Check that the equations are equivalent by writing them in slope-intercept form.

$y - 3 = -x - 1$

$y = -x + 2$

$y - 1 = -x + 1$

$y = -x + 2$

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✓ **GUIDED PRACTICE**   **for Example 3**

3. Write an equation in point-slope form of the line that passes through the points $(2, 3)$ and $(4, 4)$.

EXAMPLE 4   Solve a multi-step problem

STICKERS You are designing a sticker to advertise your band. A company charges $225 for the first 1000 stickers and $80 for each additional 1000 stickers. Write an equation that gives the total cost (in dollars) of stickers as a function of the number (in thousands) of stickers ordered. Find the cost of 9000 stickers.

Solution

STEP 1 **Identify** the rate of change and a data pair. Let C be the cost (in dollars) and s be the number of stickers (in thousands).

Rate of change, m: $80 per 1 thousand stickers
Data pair (s_1, C_1): (1 thousand stickers, $225)

STEP 2 **Write** an equation using point-slope form. Rewrite the equation in slope-intercept form so that cost is a function of the number of stickers.

$$C - C_1 = m(s - s_1) \qquad \text{Write point-slope form.}$$
$$C - 225 = 80(s - 1) \qquad \text{Substitute 80 for } m, \text{ 1 for } s_1, \text{ and 225 for } C_1.$$
$$C = 80s + 145 \qquad \text{Solve for } C.$$

STEP 3 **Find** the cost of 9000 stickers.

$$C = 80(9) + 145 = 865 \qquad \text{Substitute 9 for } s. \text{ Simplify.}$$

▶ The cost of 9000 stickers is $865.

EXAMPLE 5   Write a real-world linear model from a table

WORKING RANCH The table shows the cost of visiting a working ranch for one day and night for different numbers of people. Can the situation be modeled by a linear equation? *Explain*. If possible, write an equation that gives the cost as a function of the number of people in the group.

Number of people	4	6	8	10	12
Cost (dollars)	250	350	450	550	650

Solution

STEP 1 **Find** the rate of change for consecutive data pairs in the table.

$$\frac{350 - 250}{6 - 4} = 50, \quad \frac{450 - 350}{8 - 6} = 50, \quad \frac{550 - 450}{10 - 8} = 50, \quad \frac{650 - 550}{12 - 10} = 50$$

Because the cost increases at a constant rate of $50 per person, the situation can be modeled by a linear equation.

STEP 2 **Use** point-slope form to write the equation. Let C be the cost (in dollars) and p be the number of people. Use the data pair (4, 250).

$$C - C_1 = m(p - p_1) \qquad \text{Write point-slope form.}$$
$$C - 250 = 50(p - 4) \qquad \text{Substitute 50 for } m, \text{ 4 for } p_1, \text{ and 250 for } C_1.$$
$$C = 50p + 50 \qquad \text{Solve for } C.$$

4. WHAT IF? In Example 4, suppose a second company charges $250 for the first 1000 stickers. The cost of each additional 1000 stickers is $60.

  a. Write an equation that gives the total cost (in dollars) of the stickers as a function of the number (in thousands) of stickers ordered.

  b. Which company would charge you less for 9000 stickers?

5. MAILING COSTS The table shows the cost (in dollars) of sending a single piece of first class mail for different weights. Can the situation be modeled by a linear equation? *Explain*. If possible, write an equation that gives the cost of sending a piece of mail as a function of its weight (in ounces).

Weight (ounces)	1	4	5	10	12
Cost (dollars)	0.37	1.06	1.29	2.44	2.90

4.3 EXERCISES

SKILL PRACTICE

1. VOCABULARY Identify the slope of the line given by the equation $y - 5 = -2(x + 5)$. Then identify one point on the line.

2. ★ WRITING *Describe* the steps you would take to write an equation in point-slope form of the line that passes through the points $(3, -2)$ and $(4, 5)$.

EXAMPLE 1
for Exs. 3–13

WRITING EQUATIONS Write an equation in point-slope form of the line that passes through the given point and has the given slope m.

3. $(2, 1)$, $m = 2$ **4.** $(3, 5)$, $m = -1$ **5.** $(7, -1)$, $m = -6$

6. $(5, -1)$, $m = -2$ **7.** $(-8, 2)$, $m = 5$ **8.** $(-6, 6)$, $m = \frac{3}{2}$

9. $(-11, -3)$, $m = -9$ **10.** $(-3, -9)$, $m = \frac{7}{3}$ **11.** $(5, -12)$, $m = -\frac{2}{5}$

12. ★ MULTIPLE CHOICE Which equation represents the line that passes through the point $(-6, 2)$ and has a slope of -1?

  (A) $y + 2 = -(x + 6)$ **(B)** $y + 2 = -(x - 6)$

  (C) $y - 2 = -(x + 6)$ **(D)** $y + 1 = -2(x + 6)$

13. ERROR ANALYSIS *Describe* and correct the error in writing an equation of the line that passes through the point $(1, -5)$ and has a slope of -2.
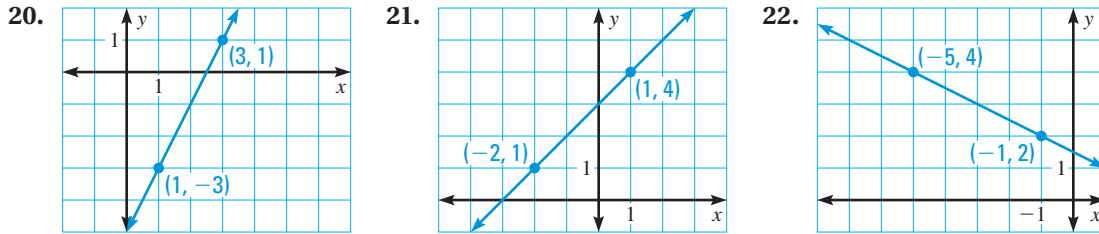
$$y - 5 = -2(x - 1)$$ ✗

GRAPHING EQUATIONS Graph the equation.

14. $y - 5 = 3(x - 1)$ **15.** $y + 3 = -2(x - 2)$ **16.** $y - 1 = 3(x + 6)$

17. $y + 8 = -(x + 4)$ **18.** $y - 1 = \frac{3}{4}(x + 1)$ **19.** $y + 4 = -\frac{5}{2}(x - 3)$

USING A GRAPH Write an equation in point-slope form of the line shown.
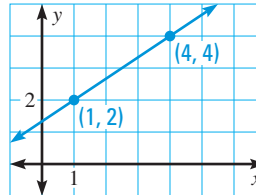
20.

21.

22.

WRITING EQUATIONS Write an equation in point-slope form of the line that passes through the given points.

23. $(7, 2)$, $(2, 12)$ **24.** $(6, -2)$, $(12, 1)$ **25.** $(-4, -1)$, $(6, -7)$

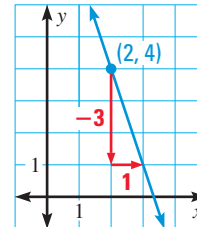**26.** $(4, 5)$, $(-4, -5)$ **27.** $(-3, -20)$, $(4, 36)$ **28.** $(-5, -19)$, $(5, 13)$

29. ERROR ANALYSIS *Describe* and correct the error in writing an equation of the line shown.

$$m = \frac{4 - 2}{4 - 1} = \frac{2}{3} \qquad y - 2 = \frac{2}{3}(x - 4)$$

30. ★ MULTIPLE CHOICE The graph of which equation is shown?

Ⓐ $y + 4 = -3(x + 2)$ **Ⓑ** $y - 4 = -3(x - 2)$

Ⓒ $y - 4 = -3(x + 2)$ **Ⓓ** $y + 4 = -3(x - 2)$

★ SHORT RESPONSE Tell whether the data in the table can be modeled by a linear equation. *Explain*. If possible, write an equation in point-slope form that relates y and x.

31.

x	2	4	6	8	10
y	−1	5	15	29	47

32.

x	1	2	3	5	7
y	1.2	1.4	1.6	2	2.4

33.

x	1	2	3	4	5
y	2	−3	4	−5	6

34.

x	−3	−1	1	3	5
y	16	10	4	−2	−8

CHALLENGE Find the value of k so that the line passing through the given points has slope m. Write an equation of the line in point-slope form.

35. $(k, 4k)$, $(k + 2, 3k)$, $m = -1$ **36.** $(-k + 1, 3)$, $(3, k + 3)$, $m = 3$

⭕ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**

37. TELEVISION In order to use an excerpt from a movie in a new television show, the television producer must pay the director of the movie $790 for the first 2 minutes of the excerpt and $130 per minute after that.

a. Write an equation that gives the total cost (in dollars) of using the excerpt as a function of the length (in minutes) of the excerpt.

b. Find the total cost of using an excerpt that is 8 minutes long.

38. ★ **SHORT RESPONSE** A school district pays an installation fee and a monthly fee for Internet service. The table shows the total cost of Internet service for the school district over different numbers of months. *Explain* why the situation can be modeled by a linear equation. What is the installation fee? What is the monthly service fee?

Months of service	2	4	6	8	10	12
Total cost (dollars)	9,378	12,806	16,234	19,662	23,090	26,518

39. COMPANY SALES During the period 1994–2004, the annual sales of a small company increased by $10,000 per year. In 1997 the annual sales were $97,000. Write an equation that gives the annual sales as a function of the number of years since 1994. Find the sales in 2000.

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40. TRAFFIC DELAYS From 1990 to 2001 in Boston, Massachusetts, the annual excess fuel (in gallons per person) consumed due to traffic delays increased by about 1.4 gallons per person each year. In 1995 each person consumed about 37 gallons of excess fuel.

a. Write an equation that gives the annual excess fuel (in gallons per person) as a function of the number of years since 1990.

b. How much excess fuel was consumed per person in 2001?
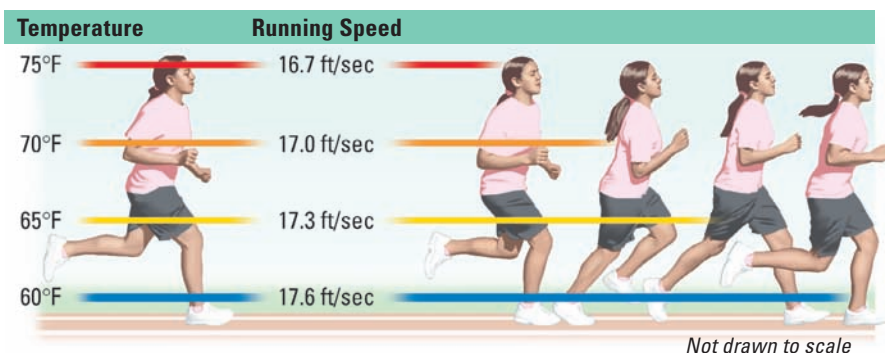
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41. ★ **EXTENDED RESPONSE** The table shows the cost of ordering sets of prints of digital photos from an online service. The cost per print is the same for the first 30 prints. There is also a shipping charge.

Number of prints	1	2	5	8
Total cost (dollars)	1.98	2.47	3.94	5.41

a. *Explain* why the situation can be modeled by a linear equation.

b. Write an equation in point-slope form that relates the total cost (in dollars) of a set of prints to the number of prints ordered.

c. Find the shipping charge for up to 10 prints.

d. The cost of 15 prints is $9.14. The shipping charge increases after the first 10 prints. Find the shipping charge for 15 prints.

42. **AQUACULTURE** Aquaculture is the farming of fish and other aquatic animals. World aquaculture increased at a relatively constant rate from 1991 to 2002. In 1994 world aquaculture was about 20.8 million metric tons. In 2000 world aquaculture was about 35.5 million metric tons.

   a. Write an equation that gives world aquaculture (in millions of metric tons) as a function of the number of years since 1991.

   b. In 2001 China was responsible for 70.2% of world aquaculture. Approximate China's aquaculture in 2001.

43. **MARATHON** The diagram shows a marathon runner's speed at several outdoor temperatures.

Temperature	Running Speed
75°F	16.7 ft/sec
70°F	17.0 ft/sec
65°F	17.3 ft/sec
60°F	17.6 ft/sec

Not drawn to scale

   a. Write an equation in point-slope form that relates running speed (in feet per second) to temperature (in degrees Fahrenheit).

   b. Estimate the runner's speed when the temperature is 80°F.

44. **CHALLENGE** The number of cans recycled per pound of aluminum recycled in the U.S. increased at a relatively constant rate from 1972 to 2002. In 1977 about 23.5 cans per pound of aluminum were recycled. In 2000, about 33.1 cans per pound of aluminum were recycled.

   a. Write an equation that gives the number of cans recycled per pound of aluminum recycled as a function of the number of years since 1972.

   b. In 2002, there were 53.8 billion aluminum cans collected for recycling. Approximately how many pounds of aluminum were collected? *Explain* how you found your answer.