

4.4 Write Linear Equations in Standard Form



Before

You wrote equations in point-slope form.

Now

You will write equations in standard form.

Why?

So you can find possible combinations of objects, as in Ex. 41.

Key Vocabulary

- standard form

Recall that the linear equation $Ax + By = C$ is in standard form, where A , B , and C are real numbers and A and B are not both zero. All linear equations can be written in standard form.

COMMON CORE

CC.9-12.A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

EXAMPLE 1 Write equivalent equations in standard form

Write two equations in standard form that are equivalent to $2x - 6y = 4$.

Solution

To write one equivalent equation, multiply each side by 2.

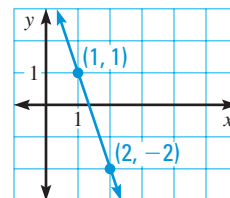
$$4x - 12y = 8$$

To write another equivalent equation, multiply each side by 0.5.

$$x - 3y = 2$$

EXAMPLE 2 Write an equation from a graph

Write an equation in standard form of the line shown.



Solution

STEP 1 Calculate the slope.

$$m = \frac{1 - (-2)}{1 - 2} = \frac{3}{-1} = -3$$

STEP 2 Write an equation in point-slope form. Use $(1, 1)$.

$$y - y_1 = m(x - x_1) \quad \text{Write point-slope form.}$$

$$y - 1 = -3(x - 1) \quad \text{Substitute 1 for } y_1, -3 \text{ for } m, \text{ and 1 for } x_1.$$

STEP 3 Rewrite the equation in standard form.

$$3x + y = 4$$

Simplify. Collect variable terms on one side, constants on the other.

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GUIDED PRACTICE for Examples 1 and 2

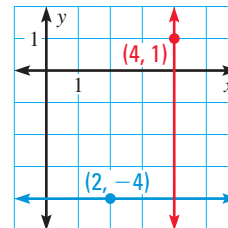
1. Write two equations in standard form that are equivalent to $x - y = 3$.
2. Write an equation in standard form of the line through $(3, -1)$ and $(2, -3)$.

HORIZONTAL AND VERTICAL LINES Recall that equations of horizontal lines have the form $y = a$. Equations of vertical lines have the form $x = b$. You cannot write an equation for a vertical line in slope-intercept form or point-slope form, because a vertical line has no slope. However, you can write an equation for a vertical line in standard form.

EXAMPLE 3 Write an equation of a line

Write an equation of the specified line.

- a. Blue line b. Red line



Solution

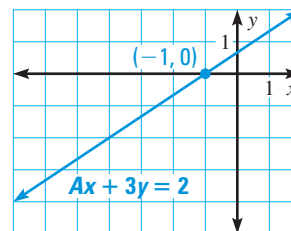
- a. The y -coordinate of the given point on the blue line is -4 . This means that all points on the line have a y -coordinate of -4 . An equation of the line is $y = -4$.
- b. The x -coordinate of the given point on the red line is 4 . This means that all points on the line have an x -coordinate of 4 . An equation of the line is $x = 4$.

ANOTHER WAY

Using the slope-intercept form to find an equation of the horizontal line gives you $y = 0x - 4$, or $y = -4$.

EXAMPLE 4 Complete an equation in standard form

Find the missing coefficient in the equation of the line shown. Write the completed equation.



Solution

STEP 1 Find the value of A . Substitute the coordinates of the given point for x and y in the equation. Solve for A .

$$Ax + 3y = 2 \quad \text{Write equation.}$$

$$A(-1) + 3(0) = 2 \quad \text{Substitute } -1 \text{ for } x \text{ and } 0 \text{ for } y.$$

$$-A = 2 \quad \text{Simplify.}$$

$$A = -2 \quad \text{Divide by } -1.$$

STEP 2 Complete the equation.

$$-2x + 3y = 2 \quad \text{Substitute } -2 \text{ for } A.$$

GUIDED PRACTICE for Examples 3 and 4

Write equations of the horizontal and vertical lines that pass through the given point.

3. $(-8, -9)$ 4. $(13, -5)$

Find the missing coefficient in the equation of the line that passes through the given point. Write the completed equation.

5. $-4x + By = 7, (-1, 1)$ 6. $Ax + y = -3, (2, 11)$



EXAMPLE 5 Solve a multi-step problem

LIBRARY Your class is taking a trip to the public library. You can travel in small and large vans. A small van holds 8 people and a large van holds 12 people. Your class could fill 15 small vans and 2 large vans.



- Write** an equation in standard form that models the possible combinations of small vans and large vans that your class could fill.
- Graph** the equation from part (a).
- List** several possible combinations.

Solution

- Write a verbal model. Then write an equation.

Capacity of small van	•	Number of small vans	+	Capacity of large van	•	Number of large vans	=	People on trip
↓		↓		↓		↓		↓
8	•	s	+	12	•	l	=	p

Because your class could fill 15 small vans and 2 large vans, use $(15, 2)$ as the s - and l -values to substitute in the equation $8s + 12l = p$ to find the value of p .

$$8(15) + 12(2) = p \quad \text{Substitute 15 for } s \text{ and 2 for } l.$$

$$144 = p \quad \text{Simplify.}$$

Substitute 144 for p in the equation $8s + 12l = p$.

- The equation $8s + 12l = 144$ models the possible combinations.

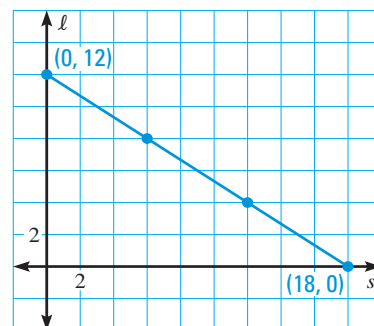
- Find the intercepts of the graph.

Substitute 0 for s . Substitute 0 for l .

$$8(0) + 12l = 144 \quad 8s + 12(0) = 144$$

$$l = 12 \quad s = 18$$

Plot the points $(0, 12)$ and $(18, 0)$. Connect them with a line segment. For this problem only nonnegative whole-number values of s and l make sense.



- The graph passes through $(0, 12)$, $(6, 8)$, $(12, 4)$, and $(18, 0)$. So, four possible combinations are 0 small and 12 large, 6 small and 8 large, 12 small and 4 large, 18 small and 0 large.

LISTING COMBINATIONS
Other combinations of small and large vans are possible. Another way to find possible combinations is by substituting values for s or l in the equation.

GUIDED PRACTICE for Example 5

- WHAT IF?** In Example 5, suppose that 8 students decide not to go on the class trip. Write an equation that models the possible combinations of small and large vans that your class could fill. List several possible combinations.

4.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**
Exs. 17 and 39

★ = **STANDARDIZED TEST PRACTICE**
Exs. 4, 30, 40, and 42

◆ = **MULTIPLE REPRESENTATIONS**
Ex. 41

SKILL PRACTICE

VOCABULARY Identify the form of the equation.

1. $2x + 8y = -3$ 2. $y = -5x + 8$ 3. $y + 4 = 2(x - 6)$

4. ★ **WRITING** Explain how to write an equation of a line in standard form when two points on the line are given.

EXAMPLE 1
for Exs. 5–10

EQUIVALENT EQUATIONS Write two equations in standard form that are equivalent to the given equation.

5. $x + y = -10$ 6. $5x + 10y = 15$ 7. $-x + 2y = 9$
8. $-9x - 12y = 6$ 9. $9x - 3y = -12$ 10. $-2x + 4y = -5$

EXAMPLE 2
for Exs. 11–22

WRITING EQUATIONS Write an equation in standard form of the line that passes through the given point and has the given slope m or that passes through the two given points.

11. $(-3, 2)$, $m = 1$ 12. $(4, -1)$, $m = 3$ 13. $(0, 5)$, $m = -2$
14. $(-8, 0)$, $m = -4$ 15. $(-4, -4)$, $m = -\frac{3}{2}$ 16. $(-6, -10)$, $m = \frac{1}{6}$
17. $(-8, 4)$, $(4, -4)$ 18. $(-5, 2)$, $(-4, 3)$ 19. $(0, -1)$, $(-6, -9)$
20. $(3, 9)$, $(1, 1)$ 21. $(10, 6)$, $(-12, -5)$ 22. $(-6, -2)$, $(-1, -2)$

EXAMPLE 3
for Exs. 23–28

HORIZONTAL AND VERTICAL LINES Write equations of the horizontal and vertical lines that pass through the given point.

23. $(3, 2)$ 24. $(-5, -3)$ 25. $(-1, 3)$
26. $(5, 3)$ 27. $(-1, 4)$ 28. $(-6, -2)$

EXAMPLE 4
for Exs. 29–36

29. **ERROR ANALYSIS** Describe and correct the error in finding the value of A for the equation $Ax - 3y = 5$, if the graph of the equation passes through the point $(1, -4)$.

$$\begin{aligned} A(-4) - 3(1) &= 5 \\ A &= -2 \end{aligned}$$

30. ★ **WRITING** The *intercept form* of the equation of a line with an x -intercept of a and a y -intercept of b is $\frac{x}{a} + \frac{y}{b} = 1$. Write the equation $2x + 3y = 12$ in intercept form. Describe your method.

COMPLETING EQUATIONS Find the missing coefficient in the equation of the line that passes through the given point. Write the completed equation.

31. $Ax + 3y = 5$, $(2, -1)$ 32. $Ax - 4y = -1$, $(6, 1)$ 33. $-x + By = 10$, $(-2, -2)$
34. $8x + By = 4$, $(-5, 4)$ 35. $Ax - 3y = -5$, $(1, 0)$ 36. $2x + By = -4$, $(-3, 7)$

37. **CHALLENGE** Write an equation in standard form of the line that passes through $(0, a)$ and $(b, 0)$ where $a \neq 0$ and $b \neq 0$.


PROBLEM SOLVING

EXAMPLE 5
for Exs. 38–41

38. **GARDENING** The diagram shows the prices of two types of ground cover plants. Write an equation in standard form that models the possible combinations of vinca and phlox plants a gardener can buy for \$300. List three of these possible combinations.



39. **NUTRITION** A snack mix requires a total of 120 ounces of some corn cereal and some wheat cereal. Corn cereal comes in 12 ounce boxes.
- The last time you made this mix, you used 5 boxes of corn cereal and 4 boxes of wheat cereal. How many ounces are in a box of wheat cereal?
 - Write an equation in standard form that models the possible combinations of boxes of wheat and corn cereal you can use.
 - List all possible combinations of whole boxes of wheat and corn cereal you can use to make the snack mix.
40. **★ SHORT RESPONSE** A dog kennel charges \$20 per night to board your dog. You can also have a doggie treat delivered to your dog for \$5. Write an equation that models the possible combinations of nights at the kennel and doggie treats that you can buy for \$100. Graph the equation. *Explain* what the intercepts of the graph mean in this situation.
41. **◆ MULTIPLE REPRESENTATIONS** As the student council treasurer, you prepare the budget for your class rafting trip. Each large raft costs \$100 to rent, and each small raft costs \$40 to rent. You have \$1600 to spend.
- Writing an Equation** Write an equation in standard form that models the possible combinations of small rafts and large rafts that you can rent.
 - Drawing a Graph** Graph the equation from part (a).
 - Making a Table** Make a table that shows several combinations of small and large rafts that you can rent.
42. **★ SHORT RESPONSE** One bus ride costs \$.75. One subway ride costs \$1.00. A monthly pass can be used for unlimited subway and bus rides and costs the same as 36 subway rides plus 36 bus rides.
- Write an equation in standard form that models the possible combinations of bus and subway rides with the same value as the pass.
 - You ride the bus 60 times in one month. How many times must you ride the subway in order for the cost of the rides to equal the value of the pass? *Explain* your answer.

43.  **GEOMETRY** Write an equation in standard form that models the possible lengths and widths (in feet) of a rectangle having the same perimeter as a rectangle that is 10 feet wide and 20 feet long. Make a table that shows five possible lengths and widths of the rectangle.
44. **CHALLENGE** You are working in a chemistry lab. You have 1000 milliliters of pure acid. A dilution of acid is created by adding pure acid to water. A 40% dilution contains 40% acid and 60% water. You have been asked to make a 40% dilution and a 60% dilution of pure acid.
- Write an equation in standard form that models the possible quantities of each dilution you can prepare using all 1000 milliliters of pure acid.
 - You prepare 700 milliliters of the 40% dilution. How much of the 60% dilution can you prepare?
 - How much water do you need to prepare 700 milliliters of the 40% dilution?

QUIZ

Write an equation in slope-intercept form of the line that passes through the given point and has the given slope m .

- $(2, 5)$, $m = 3$
- $(-1, 4)$, $m = -2$
- $(0, -7)$, $m = 5$

Write an equation in slope-intercept form of the line that passes through the given points.

- $(0, 2)$, $(9, 5)$
- $(5, 7)$, $(19, 14)$
- $(4, 24)$, $(-11, 19)$

Write an equation in (a) point-slope form and (b) standard form of the line that passes through the given points.

- $(-5, 2)$, $(-4, 3)$
- $(0, -1)$, $(-6, -9)$
- $(3, 9)$, $(1, 1)$

10. **DVDS** The table shows the price per DVD for different quantities of DVDs. Write an equation that models the price per DVD as a function of the number of DVDs purchased.

Number of DVDs purchased	1	2	3	4	5	6
Price per DVD (dollars)	20	18	16	14	12	10