Numbers can be expressed in several ways. The fraction $\frac{6}{3}$, the mixed number $1 \frac{5}{5}$, the expression $-7+9$, and $\sqrt{4}$ all describe the number 2 .

## EXAMPLE 1 Show that rational numbers are not uniquely represented

Show that $\frac{2}{3}$ is not uniquely represented as a rational number.

## Solution:

The fractions $\frac{4}{6}, \frac{6}{9}, \frac{10}{15}$, and $\frac{20}{30}$ are just some of the other rational numbers equivalent to $\frac{2}{3}$.

While rational numbers have an infinite number of equivalent fractions, all rational numbers can be written in lowest terms as one unique fraction.

## EXAMPLE2 Compare the uniqueness of prime and composite factorizations

Find unique factorizations for the numbers 11 and 28.

## Solution:

11 is prime, so its factorization $1 \times 11$ is unique.
28 is composite. It can be factored in many ways, such as $1 \times 28,2 \times 14$, and $4 \times 7$. There is only one way to break a composite number into prime factors. The prime factorization of $28,2 \times 2 \times 7$, is unique.

Examples 1 and 2 show there are many ways of describing numbers and their factorizations. Example 3 explores the concept of uniqueness as it pertains to equations of lines.

## EXAMPLE 3 Determine representations of lines that are unique

Show each of the following:
a. The point-slope form of the line with slope -1 passing through points $(3,1)$ and $(-2,6)$ is not unique.
b. The standard form of the line $x+4 y=3$ is not unique.
c. The slope-intercept form of the line $y=3 x-2$ is unique.

## Solution:

Equations are unique if there is only one way of writing the line in a specific form.
a. Two point-slope forms for this line are $y-1=-1(x-3)$ and $y-6=-1(x+2)$. Since more than one equation in point-slope form describes the line, this representation is not unique.
b. Two additional standard forms for the line $x+4 y=3$ are $2 x+8 y=6$ and $-x-4 y=-3$. Since more than one equation in standard form describes the line, this representation is not unique.

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c. All lines have exactly one slope and no more than one $y$-intercept. If the slope or $y$-intercept in $y=3 x-2$ were different, the equation would represent a different line. Suppose the slope and $y$-intercept were both doubled, changing $y=3 x-2$ to $y=6 x-4$. These equations are not the same since both lines intersect the $y$-axis at different points and have different slopes. There is only one unique way of writing an equation in slope-intercept form.

## Practice

## Write two other representations for the given number.

1. $\frac{10}{35}$
2. $\frac{9}{16}$
3. $-\frac{8}{12}$
4. $-\frac{24}{18}$
5. factors of 24
6. factors of 36
7. factors of 150
8. factors of 76

## Write the unique representation as a fraction in lowest terms.

9. $\frac{6}{15}$
10. $\frac{35}{14}$
11. $\frac{8}{18}$
12. $-\frac{56}{100}$

## Write the unique prime factorization.

13. 45
14. 60
15. 200
16. 99

## Problem Solving

17. Ticket prices to the school play are $\$ 6$ for students and $\$ 10$ for adults. The school hopes to raise $\$ 1500$ from the ticket sales. Write three equations in standard form that model this situation.
18. Jenny bought 3 shirts for the same price from a catalog company that charges a flat fee of $\$ 5$ for shipping. The total amount she paid was $\$ 41$. Meghan bought 2 of the same shirts from the same catalog and paid a total of $\$ 29$. Determine the price for each shirt. Then write three equations in point-slope form that model this situation.
19. Josh makes and sells model trains for $\$ 12$ each. His cost for materials was $\$ 60$. Write an equation in slope-intercept form that models the amount of profit Josh makes after each sale, until he uses up his materials.
20. A freight elevator lifts barrels and boxes. Each barrel weighs 40 pounds and each box weighs 25 pounds. The most weight the elevator can lift at once is 1000 pounds. Write three equations in standard form that represent the possible combinations for the most number of boxes and barrels the freight elevator can lift.
21. William has cheese and crackers for a 300 -calorie snack. Each slice of cheese has 50 calories and each cracker has 30 calories. Write three equations in standard form that show the possible number of crackers and slices of cheese that William can have for his snack.
22. Write an equation in slope-intercept form that shows all possible combinations for the number of dimes and quarters totaling $\$ 10$.
