4.5 Write Equations of Parallel and Perpendicular Lines

Before You used slope to determine whether lines are parallel.

Now You will write equations of parallel and perpendicular lines.

Why? So you can analyze growth rates, as in Ex. 33.

Key Vocabulary
- **converse**
- **perpendicular lines**
- **conditional statement**

The **converse** of a conditional statement interchanges the hypothesis and conclusion. The converse of a true statement is not necessarily true.

You have learned that the statement "If two nonvertical lines have the same slope, then they are parallel" is true. Its converse is also true.

KEY CONCEPT *For Your Notebook*

Parallel Lines

- If two nonvertical lines in the same plane have the same slope, then they are parallel.
- If two nonvertical lines in the same plane are parallel, then they have the same slope.

COMMON CORE

CC.9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

EXAMPLE 1 Write an equation of a parallel line

Write an equation of the line that passes through (−3, −5) and is parallel to the line $y = 3x - 1$.

Solution

STEP 1 **Identify** the slope. The graph of the given equation has a slope of 3. So, the parallel line through (−3, −5) has a slope of 3.

STEP 2 **Find** the y-intercept. Use the slope and the given point.

$y = mx + b$ Write slope-intercept form.

$-5 = 3(-3) + b$ Substitute 3 for m, −3 for x, and −5 for y.

$4 = b$ Solve for b.

STEP 3 **Write** an equation. Use $y = mx + b$.

$y = 3x + 4$ Substitute 3 for m and 4 for b.

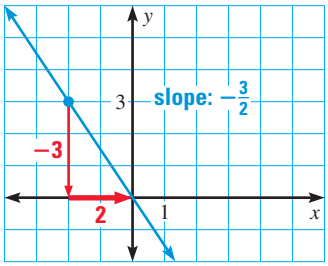
CHECK REASONABLENESS
You can check that your answer is reasonable by graphing both lines.

GUIDED PRACTICE for Example 1

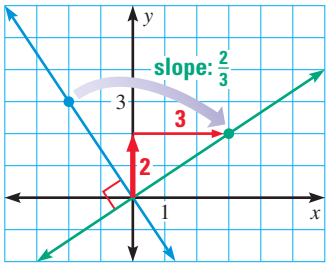
1. Write an equation of the line that passes through (−2, 11) and is parallel to the line $y = -x + 5$.

PERPENDICULAR LINES Two lines in the same plane are **perpendicular** if they intersect to form a right angle. Horizontal and vertical lines are perpendicular to each other.

Compare the slopes of the perpendicular lines shown below.



Rotate the line 90° in a clockwise direction about the origin to find a perpendicular line.



USE FRACTIONS
The product of a nonzero number m and its negative reciprocal is -1:
$m\left(-\frac{1}{m}\right) = -1$.

KEY CONCEPT *For Your Notebook*

Perpendicular Lines

- If two nonvertical lines in the same plane have slopes that are negative reciprocals, then the lines are perpendicular.
- If two nonvertical lines in the same plane are perpendicular, then their slopes are negative reciprocals.

EXAMPLE 2 Determine whether lines are parallel or perpendicular

Determine which lines, if any, are parallel or perpendicular.

Line *a*: $y = 5x - 3$ **Line *b*:** $x + 5y = 2$ **Line *c*:** $-10y - 2x = 0$

Solution

Find the slopes of the lines.

Line *a*: The equation is in slope-intercept form. The slope is 5.

Write the equations for lines b and c in slope-intercept form.

Line *b*: $x + 5y = 2$

$5y = -x + 2$

$y = -\frac{1}{5}x + \frac{2}{5}$

Line *c*: $-10y - 2x = 0$

$-10y = 2x$

$y = -\frac{1}{5}x$

▶ Lines b and c have slopes of $-\frac{1}{5}$, so they are parallel. Line a has a slope of 5, the negative reciprocal of $-\frac{1}{5}$, so it is perpendicular to lines b and c.

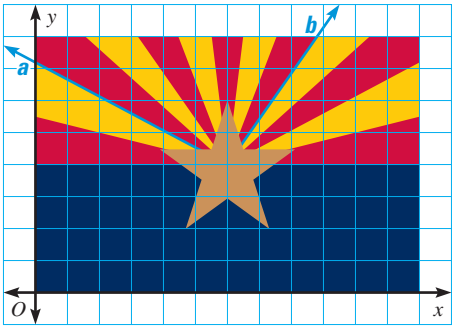
GUIDED PRACTICE for Example 2

2. Determine which lines, if any, are parallel or perpendicular.

Line *a*: $2x + 6y = -3$ **Line *b*:** $y = 3x - 8$ **Line *c*:** $-1.5y + 4.5x = 6$

EXAMPLE 3 Determine whether lines are perpendicular

STATE FLAG The Arizona state flag is shown in a coordinate plane. Lines a and b appear to be perpendicular. Are they?



Line a: $12y = -7x + 42$

Line b: $11y = 16x - 52$

Solution

Find the slopes of the lines. Write the equations in slope-intercept form.

Line a: $12y = -7x + 42$ **Line b:** $11y = 16x - 52$

$$y = -\frac{7}{12}x + \frac{42}{12} \qquad y = \frac{16}{11}x - \frac{52}{11}$$

▶ The slope of line a is $-\frac{7}{12}$. The slope of line b is $\frac{16}{11}$. The two slopes are not negative reciprocals, so lines a and b are not perpendicular.

EXAMPLE 4 Write an equation of a perpendicular line

Write an equation of the line that passes through (4, −5) and is perpendicular to the line $y = 2x + 3$.

Solution

STEP 1 **Identify** the slope. The graph of the given equation has a slope of 2. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line through (4, −5) is $-\frac{1}{2}$.

STEP 2 **Find** the y-intercept. Use the slope and the given point.

$y = mx + b$ Write slope-intercept form.

$-5 = -\frac{1}{2}(4) + b$ Substitute $-\frac{1}{2}$ for m, 4 for x, and −5 for y.

$-3 = b$ Solve for b.

STEP 3 **Write** an equation.

$y = mx + b$ Write slope-intercept form.

$y = -\frac{1}{2}x - 3$ Substitute $-\frac{1}{2}$ for m and −3 for b.

GUIDED PRACTICE for Examples 3 and 4

3. Is line a perpendicular to line b? *Justify* your answer using slopes.

Line a: $2y + x = -12$ **Line b**: $2y = 3x - 8$

4. Write an equation of the line that passes through (4, 3) and is perpendicular to the line $y = 4x - 7$.

4.5 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 19 and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 16, 17, 28, 30, 34, and 36

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: Two lines in a plane are __?__ if they intersect to form a right angle.

2. ★ **WRITING** *Explain* how you can tell whether two lines are perpendicular, given the equations of the lines.

EXAMPLE 1 for Exs. 3–11

PARALLEL LINES **Write an equation of the line that passes through the given point and is parallel to the given line.**

3. $(-1, 3)$, $y = 2x + 2$
4. $(6, 8)$, $y = -\frac{5}{2}x + 10$
5. $(5, -1)$, $y = -\frac{3}{5}x - 3$
6. $(-1, 2)$, $y = 5x + 4$
7. $(1, 7)$, $-6x + y = -1$
8. $(18, 2)$, $3y = x - 12$
9. $(-2, 5)$, $2y = 4x - 6$
10. $(9, 4)$, $y - x = 3$
11. $(-10, 0)$, $-y + 3x = 16$

EXAMPLE 2 for Exs. 12–16

PARALLEL OR PERPENDICULAR **Determine which lines, if any, are parallel or perpendicular.**

12. Line a: $y = 4x - 2$, Line b: $y = -\frac{1}{4}x$, Line c: $y = -4x + 1$
13. Line a: $y = \frac{3}{5}x + 1$, Line b: $5y = 3x - 2$, Line c: $10x - 6y = -4$
14. Line a: $y = 3x + 6$, Line b: $3x + y = 6$, Line c: $3y = 2x + 18$
15. Line a: $4x - 3y = 2$, Line b: $3x + 4y = -1$, Line c: $4y - 3x = 20$
16. ★ **MULTIPLE CHOICE** Which statement is true of the given lines?

Line a: $-2x + y = 4$ Line b: $2x + 5y = 2$ Line c: $x + 2y = 4$

(A) Lines a and b are parallel. (B) Lines a and c are parallel.

(C) Lines a and b are perpendicular. (D) Lines a and c are perpendicular.

17. ★ **SHORT RESPONSE** Determine which of the lines shown, if any, are parallel or perpendicular. *Justify* your answer using slopes.

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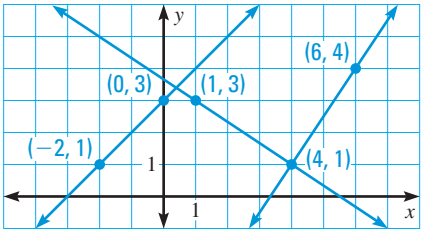


EXAMPLE 4 for Exs. 18–27

PERPENDICULAR LINES **Write an equation of the line that passes through the given point and is perpendicular to the given line.**

18. $(3, -3)$, $y = x + 5$
19. $(-9, 2)$, $y = 3x - 12$
20. $(5, 1)$, $y = 5x - 2$
21. $(7, 10)$, $y = 0.5x - 9$
22. $(-2, -4)$, $y = -\frac{2}{7}x + 1$
23. $(-4, -1)$, $y = \frac{4}{3}x + 6$
24. $(3, 3)$, $2y = 3x - 6$
25. $(-5, 2)$, $y + 3 = 2x$
26. $(8, -1)$, $4y + 2x = 12$

27. ERROR ANALYSIS *Describe* and correct the error in finding the y-intercept of the line that passes through $(2, 1)$ and is perpendicular to the line $y = -\frac{1}{2}x + 3$.

$y = mx + b$
$2 = 2(1) + b$
$0 = b$

28. ★ MULTIPLE CHOICE Which equation represents the line that passes through $(0, 0)$ and is parallel to the line passing through $(2, 3)$ and $(6, 1)$?

Ⓐ $y = \frac{1}{2}x$ Ⓑ $y = -\frac{1}{2}x$ Ⓒ $y = -2x$ Ⓓ $y = 2x$

29. REASONING Is the line through $(4, 3)$ and $(3, -1)$ perpendicular to the line through $(-3, 3)$ and $(1, 2)$? *Justify* your answer using slopes.

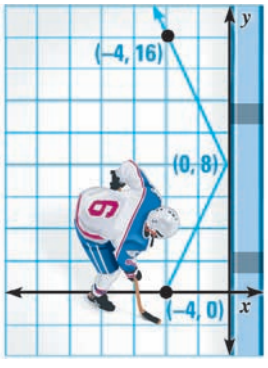
30. ★ OPEN-ENDED Write equations of two lines that are parallel. Then write an equation of a line that is perpendicular to those lines.

31. CHALLENGE Write a formula for the slope of a line that is perpendicular to the line through the points (x_1, y_1) and (x_2, y_2).

PROBLEM SOLVING

EXAMPLES 3 and 4 for Exs. 32, 34

32. HOCKEY A hockey puck leaves the blade of a hockey stick, bounces off a wall, and travels in a new direction, as shown.



a. Write an equation that models the path of the puck from the blade of the hockey stick to the wall.

b. Write an equation that models the path of the puck after it bounces off the wall.

c. Does the path of the puck form a right angle? *Justify* your answer.

33. BIOLOGY While nursing, blue whale calves can gain weight at a rate of 200 pounds per day. Two particular calves weigh 6000 pounds and 6250 pounds at birth.

a. Write equations that model the weight of each calf as a function of the number of days since birth.

b. How much is each calf expected to weigh 30 days after birth?

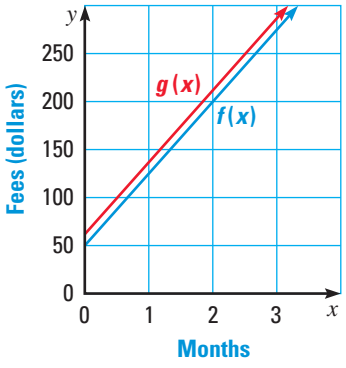
c. How are the graphs of the equations from part (a) related? *Justify* your answer.

34. ★ SHORT RESPONSE The map shows several streets in a city. Determine which of the streets, if any, are parallel or perpendicular. *Justify* your answer using slopes.

Park: $3y - 2x = 12$ Main: $y = -6x + 44$

2nd St.: $3y = 2x - 13$ Sea: $2y = -3x + 37$

35. SOFTBALL A softball training academy charges students a monthly fee plus an initial registration fee. The total amounts paid by two students are given by the functions $f(x)$ and $g(x)$ where x is the numbers of months the students have been members of the academy. The graphs of f and g are parallel lines. Did the students pay different monthly fees or different registration fees? How do you know?



36. ★ EXTENDED RESPONSE If you are one of the first 100 people to join a new health club, you are charged a joining fee of $49. Otherwise, you are charged a joining fee of $149. The monthly membership cost is $38.75.

a. Write an equation that gives the total cost (in dollars) of membership as a function of the number of months of membership if you are one of the first 100 members to join.

b. Write an equation that gives the total cost (in dollars) of membership as a function of the number of months of membership if you are *not* one of the first 100 members to join.

c. How are the graphs of these functions related? How do you know?

d. After 6 months, what is the difference in total cost for a person who paid $149 to join and a person who paid $49 to join? after 12 months?

37. CHALLENGE You and your friend have gift cards to a shopping mall. Your card has a value of $50, and your friend's card has a value of $30. If neither of you uses the cards, the value begins to decrease at a rate of $2.50 per month after 6 months.

a. Write two equations, one that gives the value of your card and another that gives the value of your friend's card as functions of the number of months after 6 months of nonuse.

b. How are the graphs of these functions related? How do you know?

c. What are the x-intercepts of the graphs of the functions, and what do they mean in this situation?