Extension

Solve Linear Inequalities by Graphing

GOAL Use graphs to solve linear inequalities.

COMMON CORE

CC.9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

You have seen how to solve linear inequalities algebraically. You can also solve linear inequalities graphically.

COMPARE FUNCTION VALUES

If you think of the equation $y = ax + b$ as a function, the solutions of $ax + b > 0$ and $ax + b < 0$ tell you where the values of the function are positive or negative.

KEY CONCEPT *For Your Notebook*

Solving Linear Inequalities Graphically

STEP 1 **Write** the inequality in one of the following forms: $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, or $ax + b \geq 0$.

STEP 2 **Write** the related equation $y = ax + b$.

STEP 3 **Graph** the equation $y = ax + b$.

- The solutions of $ax + b > 0$ are the x-coordinates of the points on the graph of $y = ax + b$ that lie above the x-axis.
- The solutions of $ax + b < 0$ are the x-coordinates of the points on the graph of $y = ax + b$ that lie below the x-axis.
- If the inequality symbol is $\leq$ or $\geq$, then the x-intercept of the graph is also a solution.

EXAMPLE 1 Solve an inequality graphically

Solve $3x + 2 > 8$ graphically.

Solution

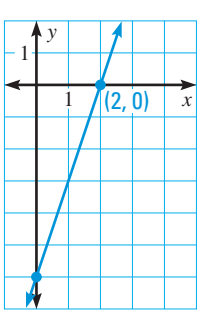
STEP 1 **Write** the inequality in the form $ax + b > 0$.

$3x + 2 > 8$ **Write original inequality.**

$3x - 6 > 0$ **Subtract 8 from each side.**

STEP 2 **Write** the related equation $y = 3x - 6$.

STEP 3 **Graph** the equation $y = 3x - 6$.



The inequality in Step 1 is in the form $ax + b > 0$, and the x-intercept of the graph in Step 3 is 2. So, $x > 2$.

▶ The solutions are all real numbers greater than 2. Check by substituting a number greater than 2 in the original inequality.

CHECK

$3x + 2 > 8$ **Write original inequality.**

$3(4) + 2 \stackrel{?}{>} 8$ **Substitute 4 for x.**

$14 > 8$ ✓ **Solution checks.**

EXAMPLE 2 Approximate a real-world solution

CELL PHONES Your cell phone plan costs $49.99 per month for a given number of minutes. Each additional minute or part of a minute costs $.40. You budgeted $55 per month for phone costs. What are the possible additional minutes x that you can afford each month?

Solution

STEP 1 **Write** a verbal model. Then write an inequality.

Rate for additional time (dollars/minute)	•	Additional time (minutes)	+	Cost of phone plan (dollars)	≤	Amount budgeted (dollars)
↓		↓		↓		↓
0.40	•	x	+	**49.99**	≤	**55**

Write the inequality in the form $ax + b \leq 0$.

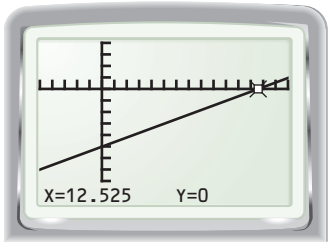
$0.40x + 49.99 \leq 55$ **Write original inequality.**

$0.40x - 5.01 \leq 0$ **Subtract 55 from each side.**

STEP 2 **Write** the related equation $y = 0.40x - 5.01$.

STEP 3 **Graph** the equation $y = 0.40x - 5.01$ on a graphing calculator.

Use the *trace* feature of the graphing calculator to find the x-intercept of the graph.



The inequality in Step 1 is in the form $ax + b \leq 0$, and the x-intercept is about 12.5. Because a part of a minute costs $.40, round 12.5 down to 12 to be sure that you stay within your budget.

▶ You can afford up to 12 additional minutes.

PRACTICE

EXAMPLES 1 and 2 for Exs. 1–4

Solve the inequality graphically.

1. $2x + 5 > 11$
2. $\frac{1}{2}x + 6 \leq 13$
3. $0.2x - 15.75 < 27$
4. **CABLE COSTS** Your family has a cable television package that costs $40.99 per month. Pay-per-view movies cost $3.95 each. Your family budgets $55 per month for cable television costs. What are the possible numbers of pay-per-view movies that your family can afford each month?