Name ______________________ Date ____________

CHAPTER 5

Compound Inequalities with No Solution or All Real Numbers as Solutions

Some compound inequalities have no solution. Other compound inequalities have the set of all real numbers as solutions.

KEY CONCEPT

Compound Inequalities with No Solution

When no values for a variable make a compound inequality true, there is no solution. We describe the solution set as the empty set $\varnothing$. When the solution set is $\varnothing$, the solution cannot be graphed.

Compound inequalities with no solutions usually come from solving compound inequalities with *and*.

EXAMPLE 1

Identify a compound inequality with no solution

Solve the inequality $8 \le x - 3 < 4$. Graph the solution, if possible.

Solution:

$$8 \le x - 3 \quad \textit{and} \quad x - 3 < 4$$

$$8 + 3 \le x - 3 + 3 \quad \textit{and} \quad x - 3 + 3 < 4 + 3$$

$$11 \le x \quad \textit{and} \quad x < 7$$

It is not possible for x to be greater than or equal to 11 *and* also be less than 7. The solution set is $\varnothing$ and cannot be graphed. ■

Notice that the compound inequality in Example 1 is false for all x values because it uses the conjunction *and*.

KEY CONCEPT

Compound Inequalities with All Real Numbers as Solutions

When all values for a variable make a compound inequality true, the solution set is the set of all real numbers. The graph of this solution set is the entire number line.

Compound inequalities with all real numbers as the solution usually come from solving compound inequalities with *or*.

EXAMPLE 2

Identify a compound inequality with all real numbers as solutions

Solve the inequality $-7 < 2p - 5$ *or* $p + 1 > 3p - 4$. Graph the solution, if possible.

Solution:

$$-7 < 2p - 5 \quad \textit{or} \quad p + 1 > 3p - 4$$

$$-2 < 2p \quad \textit{or} \quad 1 > 2p - 4$$

$$5 > 2p$$

$$-1 < p \quad \textit{or} \quad \frac{5}{2} > p$$

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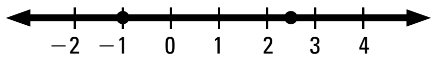
CHAPTER 5

Compound Inequalities with No Solution or All Real Numbers as Solutions *continued*

The solution set is all real numbers greater than -1 *or* less than $\frac{5}{2}$.



The graph of this solution shows both sets of numbers overlapping and continuing on infinitely in both directions. This shows that the set of all real numbers make this compound inequality true. ■

Notice that the inequality in Example 2 is true for all values of p because it uses the conjunction *or*.

Practice

Solve each inequality. Graph each solution, if possible.

1. $-5 < k + 6 < 0$

2. $15 \le 6w - 9 \le 6$

3. $4 < -2m \le 10$

4. $12 < 3x + 6$ *or* $-3x \ge -9$

5. $3 < 2g + 1 < g$

6. $1 > 4 - d$ *or* $-1(d - 6) \ge -1$

7. $5 + a < 1$ *or* $-5 > 3 - 2a$

8. $4 \ge -q - 4$ *or* $2 - 3q > 1$

9. $2(3 - b) \le b$ *or* $2b \le 9 - b$

10. $18 < 4z + 2 < -14$

11. $-1 > -7 - h > 3$

12. $j \le 3j + 2$ *or* $-2(-j - 1) \le -6$

13. Five more than y is greater than 2 *or* three less than twice y is less than or equal to 1.

14. The difference of 3 and v is greater than or equal to 4 *and* less than or equal to 3.

15. Four times the sum of 2 and n is less than -8 *and* 6 is less than twice n.

16. Seven less than 5 times c is less than 3 *or* 10 more than c is greater than -4 times c.

Problem Solving

17. The amount of money, including tips, that Gina earned working h hours last week is given by the compound inequality $300 \ge 10h + 60 \ge 400$. Discuss the number of hours that Gina could have worked.

18. Three identical blocks on one balance scale weigh less than 12 pounds *and* 8 of the same identical blocks on another scale weigh more than 40 pounds. Explain whether or not this is possible. If it is, what is the possible weight of each block? If it is not possible, explain why not.

19. The highest test score Scott can receive is 100. The sum of his first four test scores is at least 360. Find the range of scores Scott must get on his fifth test to have an average greater than 70.

20. If the length of Theresa's rectangular garden is 30 feet, she wants the perimeter to be at least 80 feet. *Or*, if the length is 20 feet, she wants the perimeter to be at most 90 feet. Describe the possible widths that Theresa's garden can be.

21. Carina's profit, in dollars, selling j pieces of jewelry in a month is shown by the inequality $100 \ge 50 + 2j > 200$. Discuss the likelihood of Carina earning this profit.