

CHAPTER
5

A Closer Look at Absolute Value

Piecewise functions define more than one rule or expression over separate intervals. The absolute value function can be seen as an example of a piecewise-defined function.

KEY CONCEPT

Definition of Absolute Value as a Piecewise-Defined Function

The definition of absolute value can be written as a piecewise-defined function.

$$y = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

You can use this definition to evaluate the absolute value of given values of x .

EXAMPLE 1 Find the absolute value of x

Find $y = |x|$ for each value of x .

a. $x = 5$

b. $x = -7$

Solution:

a. Since $5 \geq 0$, $y = |5| = 5$.

b. Since $-7 \leq 0$, $y = -(-7) = 7$. ■

The absolute value of other expressions can be derived from the above definition.

EXAMPLE 2 Find the absolute value of $-x$

Find $y = |-x|$ for each value of x .

a. $x = 6$

b. $x = -15$

Solution:

First, write $y = |-x|$ as a piecewise-defined function by substituting $-x$ for x in the above definition.

$$y = |-x| = \begin{cases} -x, & \text{when } -x \geq 0 \\ -(-x), & \text{when } -x < 0 \end{cases}$$

This can be rewritten as

$$y = |-x| = \begin{cases} -x, & \text{when } x \leq 0 \\ x, & \text{when } x > 0 \end{cases}$$

a. Since $6 > 0$, $y = |-6| = 6$.

b. Since $-15 \leq 0$,
 $y = | -(-15) | = -(-15) = 15$. ■

Piecewise-defined functions can be written for any absolute value function, as shown in Example 3.

EXAMPLE 3 Write absolute value functions as piecewise-defined functions

Write a piecewise-defined function for $y = |x + 5|$.

A Closer Look at Absolute Value *continued***Solution:**

If $x < -5$, then $x + 5$ is negative and $|x + 5| = -(x + 5)$. If $x \geq -5$, then $x + 5$ is positive and $|x + 5| = (x + 5)$.

$$|x + 5| = \begin{cases} x + 5, & \text{when } x \geq -5 \\ -(x + 5), & \text{when } x < -5 \end{cases} \blacksquare$$

Practice

Find each absolute value when $x = -10$.

1. $|x - 8|$

2. $|-5x|$

3. $|(x + 1)|$

4. $|-x - 9|$

5. $-|16 - 3x|$

Write each absolute value function as a piecewise-defined function.

6. $y = |-x|$

7. $y = -|x|$

8. $y = -|-x|$

9. $y = |4x|$

10. $y = |2 - x|$

11. $y = |2x + 3|$

12. $y = |-5x|$

13. $y = -|7 + x|$

14. $y = |-(x - 6)|$

15. $y = -|3 - 6x|$

Problem Solving

16. A quality control analyst uses the function $y = |x - 0.75|$ to measure the error when weighing packages of seeds. Write a piecewise-defined function for this error measure.
17. Shannon's car averages 25 miles per gallon. She uses the function $y = |x - 25|$ to describe the amount this mileage varies. Write a piecewise-defined function showing the amount of variation for the car's mileage per gallon.
18. During the first basketball game of the season, the captain of the team scored 12 points. The function $y = |x - 12|$ describes the variation in points scored by other members of the basketball team. Write a piecewise-defined function showing the amount of variation for the points scored by the other team members.
19. A toy company projected earnings of 5.3 hundred thousand dollars during each quarter of the year. Write a piecewise-defined function showing the amount each quarter's actual earnings varied from the projected profits.
20. Eric's gym teacher expects students to finish a race in about 45 seconds. Write a piecewise-defined function showing the amount each student's finish time varies from the gym teacher's expected finish time.
21. On a geography quiz, the average class score is an 86. Write a piecewise-defined function showing the amount each student's score varies from the average score.
22. **Challenge** Write a piecewise-defined function for $y = |x - 1| + |x + 3|$.