Name ______________________ Date ______________

CHAPTER 5

Solving Absolute Value Equations by Graphing

Absolute value equations in the form $|ax + b| = c$, where a, b, and c are constants, can be solved by graphing corresponding functions in the form $y = |ax + b| - c$ and finding the zeros. The x-values of the points where the graphs intersect the x-axis are the solutions.

KEY CONCEPT

Number of Possible Solutions to an Absolute Value Equation

An absolute value equation can have zero, one, or two solutions.

Examples of each type are shown in Example 1.

EXAMPLE 1

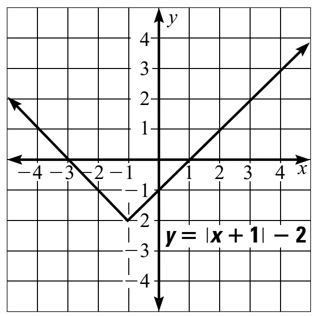
Determine the number of solutions to an absolute value equation by graphing

Determine the number of solutions to each absolute value equation by graphing.

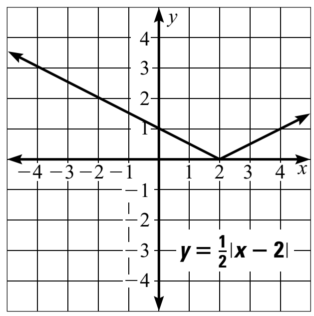
a. $|x + 1| = 2$ **b.** $\frac{1}{2}|x - 2| = 0$ **c.** $2|x - 1| = -1$

Solution:

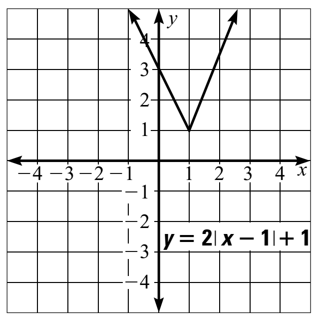
a. Graph $y = |x + 1| - 2$. There are two zeros; one at $x = 1$ and the other at $x = -3$.



b. Graph $y = \frac{1}{2}|x - 2|$. There is one zero at $x = 2$.



c. Graph $y = 2|x - 1| + 1$. There are 0 solutions. ■



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Name ______________________________ Date ______________

Solving Absolute Value Equations by Graphing *continued*

Notice the values of the constant c in each equation. In Example 1a, $c = 2$. When $c > 0$, there are 2 solutions. In Example 1b, $c = 0$. When $c = 0$, there is 1 solution. In Example 1c, $c = -1$. When $c < 0$, there are 0 solutions.

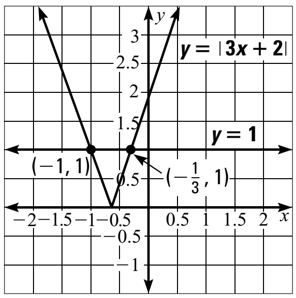
EXAMPLE 2 Find solutions to $|ax + b| = c$ by graphing $y = |ax + b|$ and $y = c$

a. Show how the graphs of $y = |3x + 2|$ and $y = 1$ can be used to find the solutions of $|3x + 2| = 1$.

b. Explain what the graphs of $y = -|x - 5|$ and $y = 2$ tell about the solutions of $y = -|x - 5| - 2$.

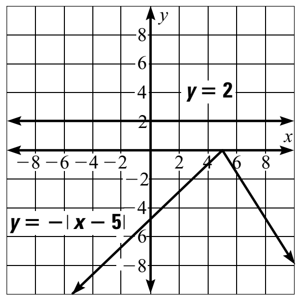
Solution:

a.



The x-coordinates of the points where these two graphs intersect are the solutions. The solutions to $|3x + 2| = 1$ are $x = -1$ and $x = -\frac{1}{3}$.

b.

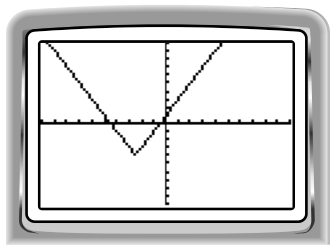


The graphs of $y = -|x - 5|$ and $y = 2$ do not intersect. Therefore, there are no solutions to the equation $y = -|x - 5| - 2$. ■

The solutions to absolute value equations can be found using a graphing calculator, as shown in Example 3.

EXAMPLE 3 Use a graphing calculator to solve absolute value equations

Solve the absolute value equation $0 = |2x + 5| - 4$ by using a graphing calculator.



Solution:

Enter the absolute value function into Y= using the absolute value function (abs) found in the MATH menu under NUM. Be sure the WINDOW is set for an appropriate domain and range.

Then graph. The graph of the absolute value function $y = |2x + 5| - 4$ is shown.

The solutions can be found using the zero command from the 2nd CALC menu on the graphing calculator.

The solutions are at $x = -0.5$ and $x = -4.5$. ■

Name ________________________ Date ____________

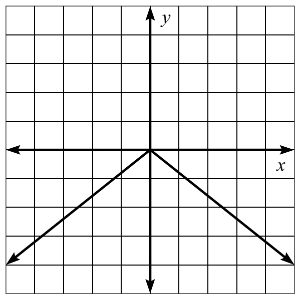
CHAPTER 5

Solving Absolute Value Equations by Graphing *continued*

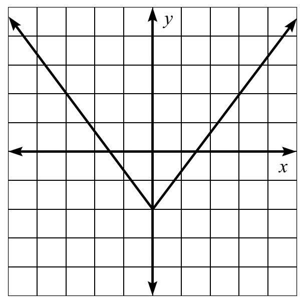
Practice

Determine the number of zeroes in each graph.

1.



2.



Determine the number of solutions to each absolute value equation.

3. $|x - 4| = 0$

4. $|4 - 3x| = 7$

5. $-|-(2 + x)| = 1$

6. $-|x - 3| = -6$

Solve each absolute value equation using a graphing calculator.

7. $0 = |x + 3|$

8. $0 = |-x - 2|$

9. $0 = -|2x|$

10. $0 = |2x - 3| + 1$

11. $0 = |x + 6| - 2$

12. $|4x + 3| = 3$

13. $|7 - x| = 0$

14. $-|2 - x| = 3$

15. $|-0.5x| = -0.5$

Problem Solving

16. Justin graphed $y = |2x - 4|$ and $y = 1$ on the same coordinate grid. Randy graphed the function $y = |2x - 4| - 1$ on another grid. How do the solutions of Justin's graphed functions compare to the zeros of Randy's graphed function?

17. Alesha graphed $y = -|x + 6|$ and $y = 1$ on the same coordinate grid. Explain how Alesha can use these graphs to find the solution to the equation $-|x + 6| = 1$. Then find the solution to the equation $-|x + 6| = 1$.

18. Draw a graph of an absolute value equation that has no solution. Explain why it has no solution.

19. Write an absolute value equation with exactly one solution. Then find its solution.

20. Mattie wrote an equation in the form $y = |ax + b| - c$ that had no solution. What possible values for a, b, and c could Mattie have used?

21. Dave wrote an equation in the form $y = |ax + b| - c$ that had two solutions. What possible values for a, b, and c could Dave have used?

22. Taylor wrote an equation in the form $y = |ax + b| - c$ that had exactly one solution. What possible values for a, b, and c could Taylor have used?