Graph Absolute Value Functions

GOAL Graph absolute value functions.

Key Vocabulary
• absolute value

The function $f(x) = |x|$ is an example of an *absolute value function* and is the parent function for all absolute value functions. You can graph absolute value functions by using a table of values, as shown below for $f(x) = |x|$.

COMMON CORE

CC.9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

KEY CONCEPT
For Your Notebook

Graph of Parent Function for Absolute Value Functions

The domain of the parent absolute value function is all real numbers. The range is $y \geq 0$.
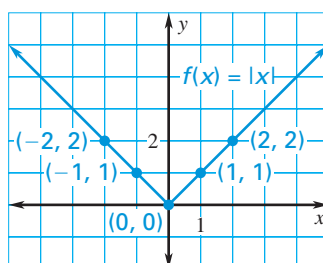
The graph consists of two rays with a common endpoint called the *vertex* of the graph. The minimum value of the function occurs at the vertex.

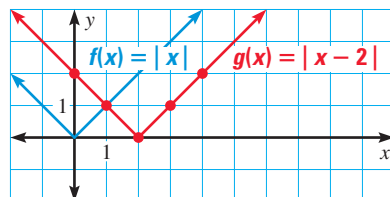
EXAMPLE 1 Graph $g(x) = |x - h|$ and $g(x) = |x| + k$

Graph each function. Compare the graph with the graph of $f(x) = |x|$.

a. $g(x) = |x - 2|$

STEP 1 Make a table of values.

x	0	1	2	3	4
g(x)	2	1	0	1	2

STEP 2 Graph the function.

APPLY TRANSFORMATIONS
The two graphs in Example 1 are translations of the graph of $f(x) = |x|$. The graph in part (a) is a horizontal translation. The graph in part (b) is a vertical translation.
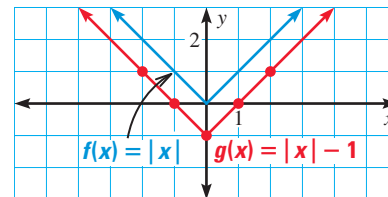

STEP 3 **Compare** the graphs of g and f. The graph of $g(x) = |x - 2|$ is 2 units to the right of the graph of $f(x) = |x|$.

b. $g(x) = |x| - 1$

STEP 1 Make a table of values.

x	−2	−1	0	1	2
g(x)	1	0	−1	0	1

STEP 2 Graph the function.

STEP 3 **Compare** the graphs of g and f. The graph of $g(x) = |x| - 1$ is 1 unit below the graph of $f(x) = |x|$.

EXAMPLE 2   **Graph $g(x) = a|x|$**

**Graph each function. Compare the graph with the graph of $f(x) = |x|$.**

a. $g(x) = 4|x|$

     STEP 1 **Make** a table of values.

x	−2	−1	0	1	2
$g(x)$	8	4	0	4	8

b. $g(x) = -0.5|x|$

     STEP 1 **Make** a table of values.

x	−4	−2	0	2	4
$g(x)$	−2	−1	0	−1	−2

APPLY TRANSFORMATIONS

The graph in part (a) of Example 2 is a vertical stretch of the graph of $f(x) = |x|$. The graph in part (b) is a vertical shrink with a reflection in the x-axis of the graph of $f(x) = |x|$.

     STEP 2 **Graph** the function.

     STEP 2 **Graph** the function.
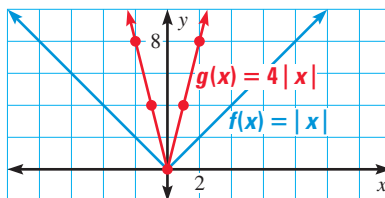
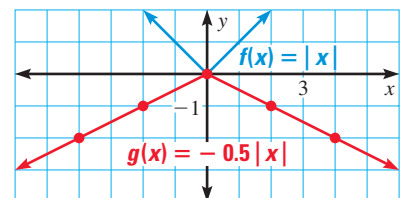   **STEP 3** **Compare** the graphs of g and f. The graph of $g(x) = 4|x|$ opens up and is narrower than the graph of $f(x) = |x|$.

     STEP 3 **Compare** the graphs of g and f. The graph of $g(x) = -0.5|x|$ opens down and is wider than the graph of $f(x) = |x|$.

KEY CONCEPT   *For Your Notebook*

Comparing Graphs of Absolute Value Functions with the Graph of $f(x) = |x|$

$g(x) = |x - h|$

If $h > 0$, the graph of g is $|h|$ units to the right of the graph of $f(x) = |x|$.

If $h < 0$, the graph of g is $|h|$ units to the left of the graph of $f(x) = |x|$.

$g(x) = |x| + k$

If $k > 0$, the graph of g is $|k|$ units above the graph of $f(x) = |x|$.

If $k < 0$, the graph of g is $|k|$ units below the graph of $f(x) = |x|$.

$g(x) = a|x|$

If $|a| > 1$, the graph of g is narrower than the graph of $f(x) = |x|$. If $0 < |a| < 1$, the graph of g is wider.

If $a > 0$, the graph of g opens up. If $a < 0$, the graph opens down.

PRACTICE

EXAMPLES
1 and 2
for Exs. 1–6

Graph the function. *Compare* the graph with the graph of $f(x) = |x|$.

1. $g(x) = |x + 3|$ **2.** $g(x) = |x| + 5$ **3.** $g(x) = |x| - 7$

4. $g(x) = 2|x|$ **5.** $g(x) = 0.6|x|$ **6.** $g(x) = -3|x|$

7. For the absolute value function $g(x) = -|x| + 1$, identify the function's domain and range, the vertex of the function's graph, and the function's minimum or maximum value.