

**CHAPTER  
5****Margins of Error**

Many measurements are not exact. Errors due to the measuring tool used or to rounding are called **margins of error**. Absolute value inequalities can be used to express the range of possible values.

If  $x$  represents the values acceptable within the margin of error of a given value, then the following absolute value inequality can be used to express the range of values:

$$|x - \text{given value}| \leq \text{margin of error}$$

**EXAMPLE 1** **Translate between measurement errors and absolute value inequalities**

Write an absolute value inequality or describe the margin of error for the situation.

- In a recent study, Camille found that 70% of students at her school play an after school sport. She determined the margin of error to be within 6%.
- To the nearest foot, the length of Damien's desk is 5 feet.
- At the supermarket, a watermelon's weight in pounds is given by  $|x - 14| \leq 3$ .

**Solution:**

- Since the range of values can be 6% more or 6% less than 70%, the absolute value inequality is  $|x - 70| \leq 6$ .
- Measurements to the nearest foot are at most 0.5 feet shorter or longer, so the absolute value inequality is  $|x - 5| < 0.5$ . Note that in this case, the inequality does not include the boundary values.
- The weight of an average watermelon at the supermarket is within 3 pounds of 14 pounds, or between 11 pounds and 17 pounds. The margin of error is 3 pounds. ■

**EXAMPLE 2** **Determine margins of error**

A gear on a machine part has a tolerance within 0.06 centimeters of 2.4 centimeters.

- Write an absolute value inequality describing the margin of error for the tolerance of this gear.
- Determine the tolerance range, in centimeters, for this gear.

**Solution:**

- $|x - 2.4| \leq 0.06$
- First rewrite the absolute value inequality as a compound inequality:

$$-0.06 \leq x - 2.4 \leq 0.06$$

Therefore,  $2.34 \leq x \leq 2.46$  or the tolerance range is between 2.34 cm and 2.46 cm. ■

The range of values within a margin of error can be graphed on a number line, as shown in Example 3.

**Margins of Error** *continued***EXAMPLE 3 Express margins of error on a number line**

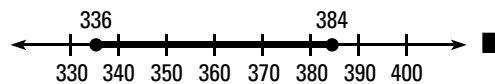
The number of bagels made each day at a bakery is within 24 of 360. Using the margin of error, find the total number of bagels that can be made each day. Then graph this range on a number line.

**Solution:**

The margin of error is 24. The absolute value inequality describing the range of values is  $|x - 360| \leq 24$ . Rewrite the absolute value inequality as a compound inequality:

$$-24 \leq x - 360 \leq 24$$

Therefore,  $336 \leq x \leq 384$ , so the range of bagels is between 336 and 384.

**Practice**

**Write the absolute value inequality describing the margin of error.**

- The length of a computer monitor, to the nearest inch, is 14 inches.
- In a survey, the number of people favoring the development of a new shopping mall was within 4 percentage points of 28%.
- In a game, points are awarded if a player's marker lands within 5 feet of a line 40 feet away.

**Describe the margin of error shown by the absolute value inequality.**

- The starting salary in dollars of employees at a retail shop is given by  $|x - 24,500| \leq 1500$ .
- The average tensile strength of a spring, in pounds, is given by  $|x - 35| < 2.5$ .
- The precision of a measurement, in centimeters, is given by  $|x - 9.6| < 0.05$ .
- The number of minutes it takes Malcolm to run a mile is given by  $|x - 7.75| \leq 0.25$ .

**Problem Solving**

- When mixing substances in the chemistry lab, Audrey's measurements can be off by no more than 10%. Describe the range of measures Audrey can make for 80 milliliters of a substance.
- To the nearest 10 millimeters, the length of a square's side is 50 millimeters. Find the possible range in area  $A$  of this square. Graph this range on a number line.
- In an experiment, Logan found that the equation  $d = 60 - 0.75p$  describes the distance  $d$ , in feet, traveled by an object weighing  $p$  pounds when projected off a platform. The margin of error is within 8 feet. What is the expected distance an object weighing 20 pounds would travel? Graph this distance range on a number line.