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CHAPTER 5

Problem Solving with Linear Equations in Two Variables

We have learned that linear inequalities result when the = sign in a linear equation is replaced with one of the inequality symbols <, >, ≤, or ≥. Here, we explore various problem situations involving each equality and inequality symbol and learn to recognize key words and phrases that help determine which symbols model these situations.

KEY CONCEPT

Meanings of Equality and Inequality Symbols

The table below shows each equality and inequality symbol, their meanings, and some key phrases that are indicators of each symbol.

Symbol	Meaning	Key Phrases
=	equals	is, exactly, the same as
<	less than	less than, fewer, below, smaller
>	greater than	greater than, more, over, above, larger, exceeds
≤	less than or equal to	less than or equal to, at most, no more than
≥	greater than or equal to	greater than or equal to, at least, no less than

Situations using inequality symbols can often appear similar. You should to be able to distinguish the differences among various situations and know when to use each type of inequality. Example 1 investigates this further.

EXAMPLE 1

Identify equality and inequality symbols modeling situations

Identify the equality or inequality symbol most appropriate for the situation.

a. Sierra studied no more than 6 hours for her exam.

b. The number of points Maria scored in this game exceeded her prior record.

c. Today's high temperature is seven degrees more than yesterday's high temperature.

d. The highest grade Timothy can receive is a 90%.

Solution:

a. Since the sentence uses the phrase, *no more than*, the appropriate symbol is ≤.

b. Since the sentence use the word *exceeded*, the appropriate symbol is >

c. Since the sentence uses the word *is*, the appropriate symbol is =

d. In this case, the highest grade has to be at most 90%. Therefore, the appropriate symbol is ≤. ■

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Notice that the situation in part c uses the = sign and not the > symbol, even though the phrase *more than* is used. Here the phrase *more than* implies addition and not the inequality >. The situation described in part d does not use any of the key words or phrases shown in the table. However, the meaning of the word *highest* is synonymous with the phrase *at most* or *no more than*. For that reason, the inequality ≤ is most appropriate.

Determining which equality and inequality symbol to use involves more than just recognizing key phrases. It requires the ability to understand the situation being modeled as a whole and to use reasoning skills.

EXAMPLE 2 Write equations and inequalities to model situations

Write an equation or inequality that best describes the situation. Then explain why the equality or inequality symbol used best models the situation.

a. Dry cleaning costs $3 for each shirt and $2 for each pair of pants. This month, Jesse budgeted no more than $24 for dry cleaning.

b. The freshman class is selling tickets to a sporting event. Student tickets cost $5 each. Adult tickets cost $8 each. The goal is to sell at least $1500 worth of tickets.

c. A crate of m magazines weighs p pounds. Each magazine weighs 0.75 pounds and the crate weighs 1.25 pounds.

Solution:

a. Let the variable s represent the number of shirts and p the number of pants. The cost for dry cleaning shirts is $3s$ and the cost for dry cleaning pants is $2p$. The inequality that best describes the situation is:

$$3s + 2p \leq 24$$

The inequality symbol ≤ is used this situation because the phrase *no more than $24* implies that amounts less than $24 or exactly $24 can be spent.

b. Let the variable s represent the number of student tickets sold and a the number of adult tickets sold. The amount from selling student tickets is $5s$ and the amount from selling adult tickets is $8a$. The inequality that best describes the situation is:

$$5s + 8a \geq 1500$$

The inequality symbol ≥ is used in this situation because the phrase *at least $1500* implies that amounts equal to $1500 or greater than $1500 are needed to meet the goal.

c. The situation can be best modeled by the equation:

$$0.75m + 1.25 = p$$

The equality symbol = is used in this situation because a linear relationship exists between the number of magazines and the total weight of the crate with magazines. No comparison is being made to suggest the total weight is less than or greater than a certain amount. ■

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Problem Solving with Linear Equations in Two Variables *continued*

EXAMPLE 3 Compare problems modeled by inequalities and equations

Carlos has at most \$20 to spend on a bouquet of flowers. Carnations cost \$1 each and roses \$2 each.

a. Describe the inequality that shows the number of carnations and roses Carlos can buy.

b. Carlos spends exactly \$20 on the bouquet. Write an equation describing the number of carnations and roses he buys.

Solution:

a. Let c be the number of carnations that Carlos buys and r the number of roses. The amount he spends on carnations is $1c$ or c, and the amount he spends on roses is $2r$. Since Carlos has *at most* \$20 to spend, he can spend exactly \$20 or any amount less than \$20. The inequality describing this is $c + 2r \leq 20$.

b. Since Carlos spends *exactly* \$20, the $\leq$ symbol in the inequality above is replaced with an $=$ symbol. The equation is $c + 2r = 20$. ■

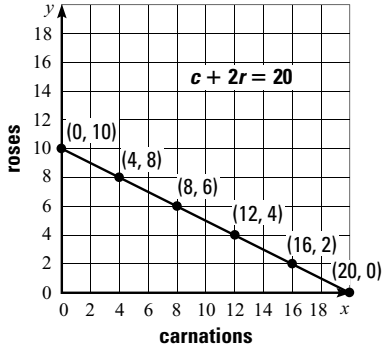
The two models in Example 3 are similar. The only difference that distinguishes *at most* \$20 from *exactly* \$20 is the inequality and equality symbols. Example 4 explores the similarities and differences between the graphs of these two models.

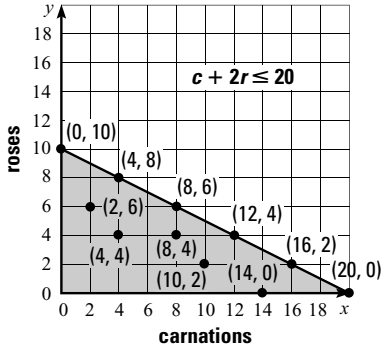
EXAMPLE 4 Compare graphs of inequalities and equations

Compare the graphs of $c + 2r = 20$ and $c + 2r \leq 20$ from Example 3. Identify at least six flower combinations that Carlos can get that satisfy each model.

Solution:

The graph of the equation $c + 2r = 20$ is a straight line containing all points on that line. The graph of the inequality $c + 2r \leq 20$ shows the same straight line but contains all points on that line *and* all points beneath that line as indicated by the shaded region.





Some flower combinations in the form (carnations, roses) satisfying $c + 2r = 20$ include (0, 10), (4, 8), (8, 6), (12, 4), (16, 2) and (20, 0).

Some flower combinations in the form (carnations, roses) satisfying $c + 2r \leq 20$ include all points on the graph of $c + 2r = 20$ as well as (2, 6), (4, 4), (8, 4), (10, 2), and (14, 0).

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Practice

Identify the equality or inequality symbol modeling each situation.

1. Dehlia babysat fewer hours last month compared to this month.
2. Victoria's monthly phone bill shows a charge of $0.15 for each minute she talks plus a service fee of $12.95.
3. The maximum weight capacity a bridge can hold is 12 tons.
4. Liam has a risk of getting sunburned if his time outdoors exceeds 45 minutes.
5. Monica plans to save at least 40% of her earnings.
6. Jasmine earns $10 an hour tutoring and $8 an hour working at the library. She wants to earn a minimum of $400 this month.

Describe all points satisfying each equation or inequality.

7. $y = 4x - 3$
8. $y \geq 4x - 3$
9. $y < 4x - 3$
10. $2x - 3y > -3$
11. $2x - 3y \leq -3$
12. $2x - 3y = -3$

Graph each equation or inequality on separate coordinate grids and compare their solutions.

13. $y < 0.5x + 2$
    $y > 0.5x + 2$
14. $y = -2x - 1$
    $y \leq -2x - 1$
15. $4x + 3y > 6$
    $4x + 3y = 6$
16. $-5x - 2y = 4$
    $-5x - 2y < 4$

Problem Solving

17. One side of a balanced scale has 3 square weights and 4 triangular weights. The other side of the scale has 4 square weights and 1 triangular weight. What equation or inequality can be used to model this situation?
18. The cost to park in a parking garage is $2.50 for the first hour and $1.50 each additional hour. Janelle plans to spend at most $10 to park for the day. Write an equation or inequality that shows the number of hours h Janelle can park in the garage and still meet her budget.
19. In a trivia game, Lyle earns 3 points for each question he answers correctly in the main round and 5 points for each question he answers correctly in the bonus round. Lyle earned at least 30 points in the game he played. Write and graph an equation or inequality showing the number of questions Lyle could have answered correctly in both rounds. Use the graph of the equation or inequality to explain whether or not Lyle could have answered 6 questions correctly in the main round and 2 questions correctly in the bonus round.