5.7 Graph Linear Inequalities in Two Variables

Before You graphed linear equations in two variables.

Now You will graph linear inequalities in two variables.

Why? So you can analyze a music competition, as in Ex. 56.

Key Vocabulary

- **linear inequality in two variables**
- **graph of an inequality in two variables**

A **linear inequality in two variables**, such as $x - 3y < 6$, is the result of replacing the $=$ sign in a linear equation with $<$, $\leq$, $>$, or $\geq$. A **solution of an inequality in two variables** x and y is an ordered pair (x, y) that produces a true statement when the values of x and y are substituted into the inequality.

EXAMPLE 1 Standardized Test Practice

Which ordered pair is *not* a solution of $x - 3y \leq 6$?

(A) $(0, 0)$ (B) $(6, -1)$ (C) $(10, 3)$ (D) $(-1, 2)$

Solution

Check whether each ordered pair is a solution of the inequality.

Test (0, 0):

$x - 3y \leq 6$ Write inequality.

$0 - 3(0) \leq 6$ Substitute 0 for x and 0 for y.

$0 \leq 6$ ✓ Simplify.

Test (6, −1):

$x - 3y \leq 6$ Write inequality.

$6 - 3(-1) \leq 6$ Substitute 6 for x and -1 for y.

$9 \leq 6$ ✗ Simplify.

So, $(0, 0)$ is a solution of $x - 3y \leq 6$ but $(6, -1)$ is *not* a solution.

▶ The correct answer is B. (A) (B) (C) (D)

GUIDED PRACTICE for Example 1

Tell whether the ordered pair is a solution of $-x + 2y < 8$.

1. $(0, 0)$ **2.** $(0, 4)$ **3.** $(3, 5)$

GRAPH OF AN INEQUALITY In a coordinate plane, the **graph of an inequality in two variables** is the set of points that represent all solutions of the inequality. The *boundary line* of a linear inequality divides the coordinate plane into two **half-planes**. Only one half-plane contains the points that represent the solutions of the inequality.

KEY CONCEPT — For Your Notebook

Graphing a Linear Inequality in Two Variables

STEP 1 **Graph** the boundary line. Use a *dashed line* for < or >, and use a *solid line* for ≤ or ≥.

STEP 2 **Test** a point not on the boundary line by checking whether the ordered pair is a solution of the inequality.

STEP 3 **Shade** the half-plane containing the point if the ordered pair is a solution of the inequality. Shade the other half-plane if the ordered pair is *not* a solution.

EXAMPLE 2 Graph a linear inequality in two variables

Graph the inequality $y > 4x - 3$.

Solution

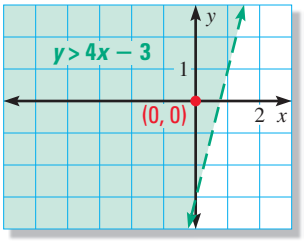
STEP 1 **Graph** the equation $y = 4x - 3$. The inequality is >, so use a dashed line.

STEP 2 **Test** (0, 0) in $y > 4x - 3$.

$$0 \stackrel{?}{>} 4(0) - 3$$

$$0 > -3 \checkmark$$

STEP 3 **Shade** the half-plane that contains (0, 0), because (0, 0) is a solution of the inequality.



EXAMPLE 3 Graph a linear inequality in two variables

Graph the inequality $x + 2y \leq 0$.

Solution

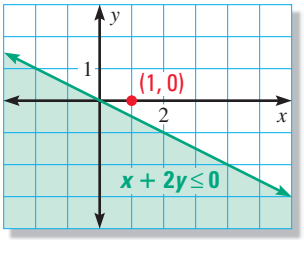
STEP 1 **Graph** the equation $x + 2y = 0$. The inequality is ≤, so use a solid line.

STEP 2 **Test** (1, 0) in $x + 2y \leq 0$.

$$1 + 2(0) \stackrel{?}{\leq} 0$$

$$1 \leq 0 \times$$

STEP 3 **Shade** the half-plane that does not contain (1, 0), because (1, 0) is *not* a solution of the inequality.



AVOID ERRORS
Be sure to test a point that is not on the boundary line. In Example 3, you can't test (0, 0) because it lies on the boundary line $x + 2y = 0$.

GUIDED PRACTICE for Examples 2 and 3

4. Graph the inequality $x + 3y \geq -1$.

LINEAR INEQUALITIES IN ONE VARIABLE The steps for graphing a linear inequality in two variables can be used to graph a linear inequality in one variable in a coordinate plane.

The boundary line for an inequality in one variable is either vertical or horizontal. When testing a point to determine which half-plane to shade, do the following:

- If an inequality has only the variable x, substitute the x-coordinate of the test point into the inequality.
- If an inequality has only the variable y, substitute the y-coordinate of the test point into the inequality.

EXAMPLE 4 Graph a linear inequality in one variable

Graph the inequality $y \geq -3$.

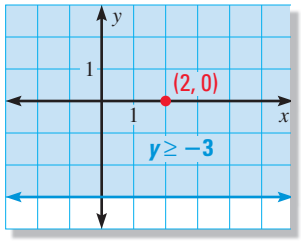
Solution

STEP 1 **Graph** the equation $y = -3$.
The inequality is $\geq$, so use a solid line.

STEP 2 **Test** (2, 0) in $y \geq -3$. You substitute only the y-coordinate, because the inequality does not have the variable x.

$$0 \geq -3 \checkmark$$

STEP 3 **Shade** the half-plane that contains (2, 0), because (2, 0) is a solution of the inequality.



EXAMPLE 5 Graph a linear inequality in one variable

Graph the inequality $x < -1$.

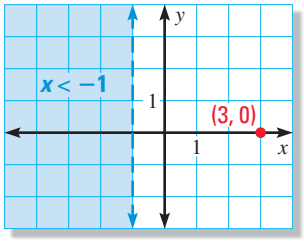
Solution

STEP 1 **Graph** the equation $x = -1$.
The inequality is $<$, so use a dashed line.

STEP 2 **Test** (3, 0) in $x < -1$. You substitute only the x-coordinate, because the inequality does not have the variable y.

$$3 < -1 \times$$

STEP 3 **Shade** the half-plane that does *not* contain (3, 0), because (3, 0) is not a solution of the inequality.



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GUIDED PRACTICE for Examples 4 and 5

Graph the inequality.

5. $y > 1$ **6.** $y \leq 3$ **7.** $x < -2$

EXAMPLE 6 Solve a multi-step problem

JOB EARNINGS You have two summer jobs at a youth center. You earn $8 per hour teaching basketball and $10 per hour teaching swimming. Let x represent the amount of time (in hours) you teach basketball each week, and let y represent the amount of time (in hours) you teach swimming each week. Your goal is to earn at least $200 per week.

- Write an inequality that describes your goal in terms of x and y.
- Graph the inequality.
- Give three possible combinations of hours that will allow you to meet your goal.

Solution

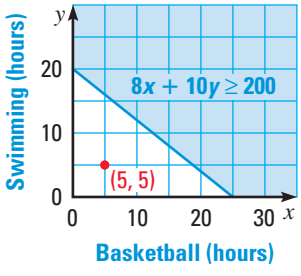
STEP 1 **Write** a verbal model. Then write an inequality.

Basketball pay rate (dollars/hour)	·	Basketball time (hours)	+	Swimming pay rate (dollars/hour)	·	Swimming time (hours)	≥	Total earnings (dollars)
↓		↓		↓		↓		↓
8	·	x	+	10	·	y	≥	200

AVOID ERRORS
The variables can't represent negative numbers. So, the graph of the inequality does not include points in Quadrants II, III, or IV.

STEP 2 **Graph** the inequality $8x + 10y \geq 200$.

First, graph the equation $8x + 10y = 200$ in Quadrant I. The inequality is ≥, so use a solid line.



Next, test (5, 5) in $8x + 10y \geq 200$:

$$8(5) + 10(5) \geq 200$$

$$90 \geq 200 \ \text{✗}$$

Finally, shade the part of Quadrant I that does not contain (5, 5), because (5, 5) is not a solution of the inequality.

STEP 3 **Choose** three points on the graph, such as (13, 12), (14, 10), and (16, 9). The table shows the total earnings for each combination of hours.

Basketball time (hours)	13	14	16
Swimming time (hours)	12	10	9
Total earnings (dollars)	224	212	218

GUIDED PRACTICE for Example 6

8. WHAT IF? In Example 6, suppose that next summer you earn $9 per hour teaching basketball and $12.50 per hour teaching swimming. Write and graph an inequality that describes your goal. Then give three possible combinations of hours that will help you meet your goal.

5.7 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 5, 19, and 57
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 16, 39, 56, 59, and 60
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The ordered pair $(2, -4)$ is a(n) __?__ of $3x - y > 7$.

2. ★ **WRITING** *Describe* the difference between graphing a linear inequality in two variables and graphing a linear equation in two variables.

EXAMPLE 1 for Exs. 3–15

CHECKING SOLUTIONS **Tell whether the ordered pair is a solution of the inequality.**

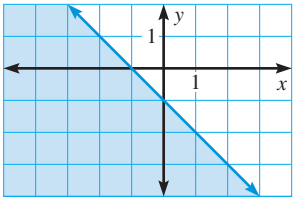
3. $x + y < -4$; $(0, 0)$
4. $x - y \leq 5$; $(8, 3)$
5. $y - x > -2$; $(-1, -4)$
6. $2x + 3y \geq 14$; $(5, 2)$
7. $4x - 7y > 28$; $(-2, 4)$
8. $-3y - 2x < 12$; $(5, -6)$
9. $2.8x + 4.1y \leq 1$; $(0, 0)$
10. $0.5y - 0.5x > 3.5$; $(6, 2)$
11. $x \geq -3$; $(-4, 0)$
12. $y \leq 8$; $(-9, -7)$
13. $\frac{3}{4}x - \frac{1}{3}y < 6$; $(-8, 12)$
14. $\frac{2}{5}x + y \geq 2$; $(1, 2)$

15. ★ **MULTIPLE CHOICE** Which ordered pair is *not* a solution of $x + 5y < 15$?

(A) $(-1, -3)$ (B) $(-1, 3)$ (C) $(1, 3)$ (D) $(3, 2)$

EXAMPLES 2, 3, 4, and 5 for Exs. 16–38

16. ★ **MULTIPLE CHOICE** The graph of which inequality is shown?



(A) $x + y \leq -1$ (B) $x + y \geq -1$

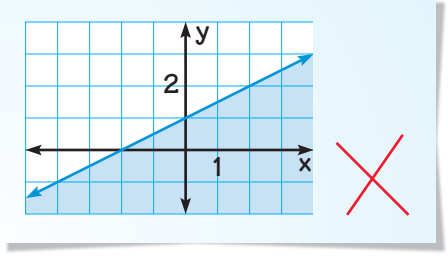
(C) $x - y \leq -1$ (D) $x - y \geq -1$

GRAPHING INEQUALITIES **Graph the inequality.**

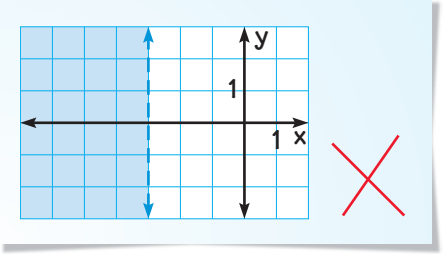
17. $y > x + 3$
18. $y \leq x - 2$
19. $y < 3x + 5$
20. $y \geq -2x + 8$
21. $x + y < -8$
22. $x - y \leq -11$
23. $x + 8y > 16$
24. $5x - y \geq 1$
25. $2(x + 2) > 7y$
26. $y - 4 < x - 6$
27. $-4y \leq 16x$
28. $6(2x) \geq -24y$
29. $y < -3$
30. $x \geq 5$
31. $x > -2$
32. $y \leq 4$
33. $3(x - 2) > y + 8$
34. $x - 4 \leq -2(y + 6)$
35. $\frac{1}{2}(x + 2) + 3y < 8$
36. $2(x + 1) \geq \frac{1}{4}y - 1$

ERROR ANALYSIS ***Describe*** **and correct the error in graphing the inequality.**

37. $2y - x \geq 2$



38. $x \leq -3$



39. ★ **WRITING** Can you use (0, 0) as a test point when graphing $2x > -5y$? *Explain* your reasoning.

TRANSLATING SENTENCES **Write the verbal sentence as an inequality. Then graph the inequality.**

40. Four less than x is greater than or equal to y.

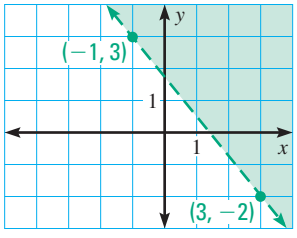
41. The product of -2 and y is less than or equal to the sum of x and 6.

42. The quotient of y and 2 is greater than the difference of 7 and x.

43. The sum of x and the product of 4 and y is less than -3.

USING A GRAPH **Write an inequality of the graph shown.**

44.



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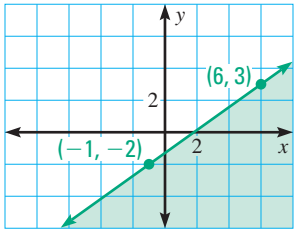

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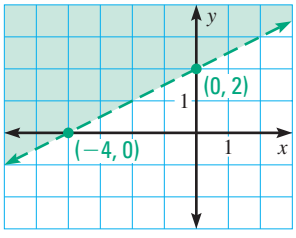


WRITING INEQUALITIES **Write an inequality whose graph contains only the points in the given quadrants.**

47. Quadrants I and II

48. Quadrants II and III

49. Quadrants III and IV

50. Quadrants I and IV

CHALLENGE **In Exercises 51 and 52, write and graph an inequality whose graph is described by the given information.**

51. The points (2, 5) and $(-3, -5)$ lie on the boundary line. The points (6, 5) and $(-2, -3)$ are solutions of the inequality.

52. The points $(-7, -16)$ and (1, 8) lie on the boundary line. The points $(-7, 0)$ and (3, 14) are *not* solutions of the inequality.

PROBLEM SOLVING

EXAMPLE 6 for Exs. 53–57

53. **BOBSLEDS** In a two-man bobsled competition, the sum of the weight x (in pounds) of the bobsled and the combined weight y (in pounds) of the athletes must not exceed 860 pounds. Write and graph an inequality that describes the possible weights of the bobsled and the athletes. Identify and interpret one of the solutions.

54. **ELEVATORS** The number y of passengers riding an elevator can be no greater than the elevator's maximum weight capacity x (in pounds) divided by 150. Write and graph an inequality that relates the number of passengers to the maximum weight capacity. Identify and interpret one of the solutions.

55. **MULTIPLE REPRESENTATIONS** You tutor Spanish for $15 per hour and French for $10 per hour. You want to earn at least $100 per week.

a. Writing an Inequality Write an inequality that describes your goal in terms of hours spent tutoring Spanish and hours spent tutoring French.

b. Drawing a Graph Graph the inequality. Then give three possible combinations of hours that meet your goal.

c. Making a Table Make a table that gives the amount of money that you will earn for each combination of hours given in part (b).

56. ★ **MULTIPLE CHOICE** To compete in a piano competition, you need to perform two musical pieces whose combined duration is no greater than 15 minutes. Which inequality describes the possible durations x and y (in minutes) of the pieces?

Ⓐ $x + y < 15$ Ⓑ $x + y \leq 15$ Ⓒ $x + y > 15$ Ⓓ $x + y \geq 15$

57. **MULTI-STEP PROBLEM** You are making muffins and loaves of bread for a bake sale. You need $\frac{1}{6}$ batch of batter per muffin and $\frac{1}{2}$ batch of batter per loaf of bread. You have enough ingredients to make up to 12 batches of batter.

a. Write and graph an inequality that describes the possible combinations of muffins m and loaves ℓ of bread that you can make.

b. You make 4 loaves of bread. What are the possible numbers of muffins that you can make?

58. **NUTRITION** A nutritionist recommends that the fat calories y consumed per day should be at most 30% of the total calories x consumed per day.

a. Write and graph an inequality that relates the number of fat calories consumed to the total calories consumed.

b. Use the nutrition labels below. You normally consume 2000 calories per day. So far today you have eaten 6 crackers and 1 container of yogurt. What are the possible additional fat calories that you can consume today?

59. ★ **SHORT RESPONSE** You need to bring a duffel and a bedroll for a trip in the mountains. The sum of the weight x (in pounds) of the duffel and the weight y (in pounds) of the bedroll cannot exceed 30 pounds.

a. Graph and Apply Write and graph a linear inequality that describes the possible weights of the duffel and bedroll. Then give three possible combinations of weights of the duffel and bedroll.

b. Interpret Are (0, 30) and (30, 0) solutions of the inequality in part (a)? Do these ordered pairs make sense for this situation? *Explain*.

60. ★ **EXTENDED RESPONSE** A financial advisor suggests that if a person is an aggressive investor, the percent y of money that the person invests in stocks should be greater than the difference of 110 and the person's age x.

a. Graph Write and graph a linear inequality that relates the percent of money invested in stocks to an aggressive investor's age.

b. Calculate If an aggressive investor is 30 years old, what are the possible percents that the investor can invest in stocks? *Explain* your answer.

c. Justify Are there any ages for which none of the solutions of the inequality makes sense for this situation? *Justify* your answer.

61. **CHALLENGE** The formula $m = dV$ gives the mass m of an object in terms of the object's density d and its volume V. Water has a density of 1 gram per cubic centimeter. An object immersed in water will sink if its density is greater than the density of water. An object will float in water if its density is less than the density of water.

a. For an object that sinks, write and graph an inequality that relates its mass (in grams) to its volume (in cubic centimeters). For an object that floats, write and graph an inequality that relates its mass (in grams) to its volume (in cubic centimeters).

b. A cylindrical can has a radius of 5 centimeters, a height of 10 centimeters, and a mass of 2119.5 grams. Will the can sink or float in water? *Explain* your answer.

QUIZ

Solve the equation.

1. $|x| = 5$

2. $|c - 8| = 24$

3. $-2|r - 5| = -6$

Solve the inequality. Graph your solution.

4. $|y| > 4$

5. $|2t - 5| < 3$

6. $4|3s + 7| - 5 \geq 7$

Graph the inequality.

7. $x + y \geq 3$

8. $\frac{5}{7}x < 10$

9. $2y - x \leq 8$

MIXED REVIEW of Problem Solving

MATHEMATICAL PRACTICES Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** You gathered 36 apples from your backyard apple tree in order to make apple pies and applesauce. You use 7 apples to make one apple pie and 5 apples to make one pint of applesauce.
   - **a.** Write an inequality that describes the possible numbers of apple pies and pints of applesauce that you can make.
   - **b.** Graph the inequality.
   - **c.** Give three possible combinations of apple pies and pints of applesauce that you can make.

2. **SHORT RESPONSE** You are scooping ice cream as part of your training at an ice cream shop. The weight of a scoop must be 4 ounces with an absolute deviation of at most 0.5 ounce.
   - **a.** Write an inequality to find the possible weights (in ounces) of each scoop.
   - **b.** You make 10 scoops. You can start working at the shop if at least 80% of the scoops meet the weight requirement. The list shows the weights (in ounces) of your scoops.

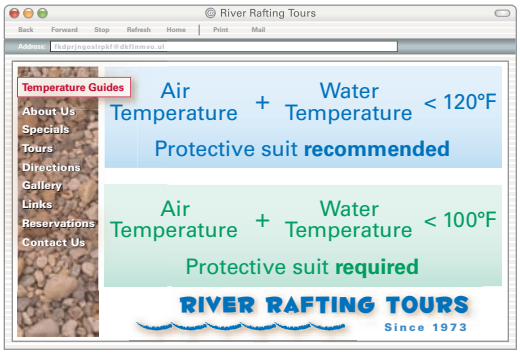
     3.8, 4.2, 3.9, 4.5, 3.7, 4.6, 4.1, 3.3, 4.3, 4.2

     Can you start working at the shop? *Explain* your reasoning.

3. **GRIDDED ANSWER** You will be making a presentation in your history class. Your teacher gives you a time limit of 15 minutes with an absolute deviation of 1.5 minutes. What is the maximum possible duration (in minutes) of your presentation?

4. **OPEN-ENDED** *Describe* a real-world situation that can be modeled by the equation $|x - 50| = 10$. *Explain* what the solution of the equation means in this situation.

5. **EXTENDED RESPONSE** A tour operator recommends that a river rafter wear a protective suit under the temperature conditions described below.



   - **a.** Write and graph an inequality that describes the possible air temperatures and water temperatures for which a protective suit is recommended.
   - **b.** If the water temperature is 40°F, for which air temperatures is a protective suit recommended?
   - **c.** How would you change the graph in part (a) in order to describe the situations in which a protective suit is required? *Explain* your answer.

6. **MULTI-STEP PROBLEM** You are buying a new cell phone and see eight phones listed on a website. The prices of the phones are shown.

   $139, $249, $229, $199, $179, $359, $199, $209

   - **a.** Find the mean price of the phones.
   - **b.** You are willing to purchase a phone that has the mean price with an absolute deviation of at most $50. Write and solve an inequality to find the prices of phones that you will consider.
   - **c.** How many of the phones on the website will you consider buying?