LESSON 5

Converting Measurements

GOAL Convert units within systems and between systems, including distances, rates, derived units, and geometric units.

To convert measurements within the U.S. customary system of measurement, you need to know the basic equivalences among the most common units. The table summarizes these equivalences.

To perform a conversion, you can multiply by one or more ratios of equivalent measures, called *conversion factors*. By carrying units through a conversion, you can perform *unit analysis* (also known as *dimensional analysis*) to make sure that you are using the correct conversion factor(s). Note that in this lesson you may consider all given measurements to be exact.

Converting Within the U.S. Customary System	
Length	12 inches (in.) = 1 foot (ft)
	3 feet = 1 yard (yd)
	5280 feet = 1 mile (mi)
Weight	16 ounces (oz) = 1 pound (lb)
	2000 pounds = 1 ton
Capacity	8 fluid ounces (fl oz) = 1 cup (c)
	2 cups = 1 pint (pt)
	2 pints = 1 quart (qt)
	4 quarts = 1 gallon (gal)

EXAMPLE 1 Converting Customary Units

Convert 12,672 feet to miles.

SOLUTION

Write a conversion factor equal to 1 using the equivalence between feet and miles. The unit you are converting to should appear in the numerator. The unit you are converting from should appear in the denominator. So, the conversion factor is $\frac{1 \text{ mi}}{5280 \text{ ft}}$.

Multiply the given measurement by the conversion factor.

$$12{,}672 \text{ ft} = 12{,}672 \cancel{\text{ft}} \cdot \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} = \frac{12{,}672}{5280} \text{ mi} = 2.4 \text{ mi}$$

$$12{,}672 \text{ ft} = 2.4 \text{ mi}$$

Converting within the metric system is often simpler than converting within the U.S. customary system because units are related by powers of 10, as shown at the right.

Converting Within the Metric System	
Length	1 kilometer (km) = 1000 meters (m)
	1 meter = 100 centimeters (cm)
	1 centimeter = 10 millimeters (mm)
Mass	1 kilogram (kg) = 1000 grams (g)
	1 gram = 1000 milligrams (mg)
Capacity	1 kiloliter (kL) = 1000 liters (L)
	1 liter = 1000 milliliters (mL)

EXAMPLE 2 Converting Metric Units

A carpenter has a piece of wood that is 3.9 meters long. The carpenter cuts off a piece that is 95.5 centimeters long. What is the length of the remaining piece of wood?

SOLUTION

Convert 95.5 centimeters to meters. The conversion factor is $\frac{1\text{ m}}{100\text{ cm}}$.

$$95.5\text{ cm} = 95.5\text{ cm} \cdot \frac{1\text{ m}}{100\text{ cm}} = \frac{95.5}{100}\text{ m} = 0.995\text{ m}$$

Subtract to find the length of the remaining piece of wood.

$$3.9\text{ m} - 0.955\text{ m} = 2.945\text{ m}$$

The length of the remaining piece of wood is 2.945 m (or 294.5 cm).

CHECK Examples 1 and 2

Perform the indicated conversion.

1. 6.5 c to fluid ounces **2.** 44.8 oz to pounds **3.** 364 mg to grams

4. A student has 135 mL of iodine in a beaker. He adds 0.44 L of iodine to the beaker. What is the total amount of iodine in the beaker?

The table below shows some of the key relationships for converting between the U.S. customary and metric systems.

Converting Between Systems		
Length	**Weight/Mass**	**Capacity**
1 in. = 2.54 cm 1 ft ≈ 0.305 m 1 yd ≈ 0.914 m 1 mi ≈ 1.61 km	1 oz ≈ 28.4 g 1 lb ≈ 0.454 kg	1 fl oz ≈ 29.6 mL 1 qt ≈ 0.946 L 1 gal ≈ 3.79 L

EXAMPLE 3 Converting Between Systems

Convert 4.2 gal to milliliters.

SOLUTION

Multiply by the conversion factor $\frac{3.79\text{ L}}{1\text{ gal}}$ to convert gallons to liters.

Multiply by the conversion factor $\frac{1000\text{ mL}}{1\text{ L}}$ to convert liters to milliliters.

$$4.2\text{ gal} \approx 4.2\text{ gal} \cdot \frac{3.79\text{ L}}{1\text{ gal}} \cdot \frac{1000\text{ mL}}{1\text{ L}} = \frac{4.2 \cdot 3.79 \cdot 1000}{1}\text{ mL} = 15{,}918\text{ mL}$$

$$4.2\text{ gal} \approx 15{,}900\text{ mL}$$

CHECK Example 3

Perform the indicated conversion.

5. 0.294 kg to pounds **6.** 2 km to miles **7.** 2 km to feet

In Example 3, two conversion factors were used to perform the conversion. Notice that when you use multiple conversion factors, all of the units in the factors should divide out, except for the unit you are converting to.

Derived units are units that are obtained by combining basic units of length, mass, capacity, and time, such as mi/h or g/cm^3. Example 4 shows how to use a series of conversion factors to convert derived units.

EXAMPLE 4 Converting Derived Units

The density of gold is 19.32 g/cm^3. Convert the density of gold to pounds per cubic inch.

SOLUTION

Two conversion factors that relate grams and pounds are $\frac{1\text{ lb}}{454\text{ g}}$ and $\frac{454\text{ g}}{1\text{ lb}}$.

Two conversion factors that relate centimeters and inches are $\frac{2.54\text{ cm}}{1\text{ in.}}$ and $\frac{1\text{ in.}}{2.54\text{ cm}}$. So, two conversion factors that relate cubic centimeters and cubic inches are $\left(\frac{2.54\text{ cm}}{1\text{ in.}}\right)^3 = \frac{(2.54\text{ cm})^3}{(1\text{ in.})^3}$ and $\left(\frac{1\text{ in.}}{2.54\text{ cm}}\right)^3 = \frac{(1\text{ in.})^3}{(2.54\text{ cm})^3}$.

Now multiply 19.32 g/cm^3 by a series of conversion factors so that like units divide out and you are left with $lb/in.^3$

$$19.32\text{ g/cm}^3 \approx \frac{19.32\text{ g}}{1\text{ cm}^3} \cdot \frac{1\text{ lb}}{454\text{ g}} \cdot \frac{(2.54\text{ cm})^3}{(1\text{ in.})^3}$$

$$= \frac{19.32\,\cancel{\text{g}}}{1\,\cancel{\text{cm}^3}} \cdot \frac{1\text{ lb}}{454\,\cancel{\text{g}}} \cdot \frac{(2.54)^3\,\cancel{\text{cm}^3}}{1^3\text{ in.}^3}$$

$$= \frac{(19.32)\,(2.54)^3\text{ lb}}{454\text{ in.}^3}$$

$$\approx 0.697\text{ lb/in.}^3$$

The density of gold is approximately 0.697 $lb/in.^3$

CHECK Example 4

8. The density of silver is 10,490 kg/m^3. Convert the density of silver to pounds per cubic foot.

Recall that a rate is a comparison of two quantities with different units. Rates are usually expressed with derived units. For example, a rate of 80 feet per 4 seconds can be written as 20 ft/sec. As shown in Example 5, you can use dimensional analysis to help you solve problems involving rates.

EXAMPLE 5 Solving a Problem Involving Rates

Two billboards at the side of a highway are 175 meters apart. A car passes one of the billboards, and 7 seconds later it passes the second billboard. Is the car going faster or slower than the 60 mi/h speed limit?

SOLUTION

The car's rate is 175 meters per 7 seconds or $\frac{175}{7} = 25$ m/sec. Convert the rate to miles per hour.

The conversion tables do not give the relationship between meters and miles. Use a conversion factor to convert meters to kilometers, and then use another conversion factor to convert kilometers to miles.

$$25 \text{ m/sec} \approx \frac{25 \text{ m}}{1 \text{ sec}} \cdot \frac{1 \text{ km}}{1000 \text{ m}} \cdot \frac{1 \text{ mi}}{1.61 \text{ km}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ h}}$$

$$= \frac{25 \cdot 60 \cdot 60 \text{ mi}}{1000 \cdot 1.61 \text{ h}}$$

$$\approx 55.9 \text{ mi/h}$$

The car is going slower than the 60 mi/h speed limit.

CHECK Example 5

9. A car that is sold in Canada is able to travel 500 kilometers on 36 liters of gas. An advertisement for the same car in the United States claims that the car's fuel efficiency is more than 35 mi/gal. Do you agree or disagree? Explain.

EXERCISES

Perform the indicated conversion.

1. 114 in. to feet

2. 3.8 tons to pounds

3. 300 c to gallons

4. 8 mi to yards

5. 0.4 oz to pounds

6. 160 fl oz to gallons

7. 0.0043 kg to milligrams

8. 1258 cm to meters

9. 0.0005 L to milliliters

10. 7.29 m to centimeters

11. 8 g to kilograms

12. 105 mm to meters

13. 17 yd to meters

14. 4.5 qt to milliliters

15. 0.03 mi to meters

16. 120 cm to feet

17. 35 kg to ounces

18. 325 fl oz to liters

19. A large pot contains $2\frac{1}{2}$ gallons of chicken broth. A cook pours out 12 cups of broth for a recipe. How many gallons of chicken broth are left in the pot? How many cups of chicken broth are left in the pot?

20. A hiking trail is 5.3 kilometers long. Gary starts hiking at the beginning of the trail and takes a photograph every 400 meters. He stops to rest after the seventh photograph. How much farther must he hike to complete the trail?

21. The table shows the average length of some lizards that are commonly found in Missouri.

Lizards of Missouri	
Name	**Average length**
Texas Horned Lizard	3 in.
Northern Fence Lizard	127 mm
Ground Skink	10.2 cm
Racerunner	$\frac{2}{3}$ ft

a. List the lizards in order from shortest to longest.

b. To the nearest centimeter, what is the range of the lengths of these lizards?

22. A rectangular poster has an area of 864 $in.^2$ The poster is 81 centimeters long. To the nearest centimeter, what is the width of the poster?

23. The density of lead is 11.36 g/cm^3. Convert the density of lead to kilograms per cubic meter and to pounds per cubic foot.

24. During takeoff, the space shuttle accelerates at a rate of 29 m/sec^2. (This means that the shuttle's speed increases by 29 m/sec every second.) Acceleration due to gravity, denoted by *g*, is approximately 32 ft/sec^2. An astronaut claims that during takeoff the acceleration of the space shuttle is about 3*g*. Do you agree? Explain.

25. The Guadalupe River in Texas usually flows at a rate of about 200 ft^3/sec. Express the river's flow rate in cubic meters per minute.

26. A high-speed elevator can travel 1010 meters in one minute. What is the elevator's rate in feet per second?

27. A horse runs one furlong in 12 seconds. What is the horse's speed in miles per hour? (*Hint*: One furlong is $\frac{1}{8}$ mile.)

28. A bus is traveling at a constant speed of 36 mi/h. To the nearest second, how long does it take the bus to travel the length of a city block that is 320 feet long?

29. A student gathers distance and time data on the motion of a jaguar running at top speed. The student calculates that the jaguar's top speed is 30 m/min. Do you think the student's calculation is correct? Explain why or why not, and explain your method.

30. A person-day is a unit of measure based on the amount of work done by an average worker in one 8-hour work day. At the widget factory, it takes 30 person-days to produce 52 cases of widgets. A case contains 12 widgets. About how long does it take an average worker to produce 6 widgets?

31. The volume of a block of clay is 216 $in.^3$ How many pieces of clay with a volume of 500 cm^3 can a sculptor make from the block? Assuming she makes the maximum number of such pieces, what is the volume of the remaining clay?