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CHAPTER 6

Exploring Systems of Three Linear Equations

Solving a system of three linear equations in three variables is hard to do graphically, even with a graphing calculator. However, graphing calculators provide another means of solving linear systems by using matrices. This method can be used with systems of two equations in two unknowns, three equations in three unknowns, four equations in four unknowns, and so on.

Recall from the Extension on pages 94 and 95 of the textbook, that an $n \times m$ matrix is a rectangular block of numbers that has n rows and m columns. The numbers n and m are called the **dimensions** of the matrix.

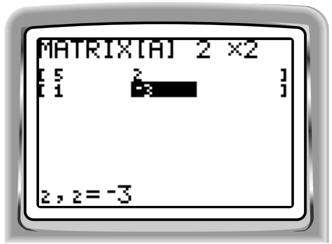
To solve a linear system of equations, you will use a graphing calculator to multiply the **inverse** of a matrix and another matrix. The inverse of an $n \times n$ matrix A is an $n \times n$ matrix, denoted A^{-1}, such that when the two matrices are multiplied under matrix multiplication, the resulting matrix is an $n \times n$ matrix with 1's along the diagonal from the upper left to lower right and 0's everywhere else. This matrix is called an **identity matrix.** We will not go into the details of matrix multiplication here, but we will use a graphing calculator to perform this operation.

EXAMPLE 1 Solve a linear system using matrices

On a graphing calculator, use matrices to solve the system $\begin{cases} 5x + 2y = 6 \\ x - 3y = -5 \end{cases}$.

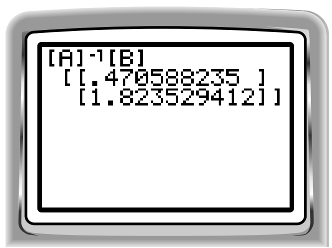
Solution:

Make sure the equations are in standard form, with variables on the left side and constants on the right, and zero coefficients included as needed. On the calculator, set matrix A as a 2×2 matrix and enter the coefficients of the system—one row for each equation, one column for each variable. (Consult the manual for your particular calculator as necessary.)



Now set matrix B as a 2×1 matrix and enter the constants from the right side of each equation, making sure to keep them in correct order.

Finally, go to the main screen and multiply the inverse of matrix A by matrix B, using the x^{-1} key on your calculator to indicate the matrix inverse operation.



The resulting matrix gives the solution values. Note that these values, with $x \approx 0.47$ and $y \approx 1.8$, match the solution found by graphing in the Graphing Calculator Activity on page 434 of the textbook. ■

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CHAPTER 6

Exploring Systems of Three Linear Equations *continued*

This method works for larger systems, such as three equations in three variables, as long as the number of variables equals the number of equations and the system has a single, well-defined solution.

KEY CONCEPT

Solving a Linear System Using Matrices

In general, to solve a linear system with n equations and n variables, multiply the inverse of the $n \times n$ matrix of coefficients by the $n \times 1$ matrix of constants. The result is the matrix of solution values.

EXAMPLE 2

Solve a linear system using matrices

On a graphing calculator, use matrices to solve the following system:

$$2x + 3y + z = 6$$
$$-x + 2y + 3z = 4$$
$$3x - z = 2$$

Solution:

Using a graphing calculator, enter $A = \begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$ Then compute $A^{-1}B$, which results in the matrix $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Therefore, the solution is $x = 1, y = 1, z = 1$. ■

Sometimes the calculator will display an error message like "SINGULAR MAT". When this message occurs, it means that the system has either infinitely many solutions or no solution at all. A message like "INVALID DIM" or "DIM MISMATCH" means that one or both matrices have been entered with incorrect dimensions.

Practice

Use the matrix method to verify the solutions obtained by the graphing method for each of the Practice Exercises in the Graphing Calculator Activity on page 434 of the textbook. Then solve the linear systems.

1. $5x + y = -4$
$x - y = -2$

2. $y = x + 4$
$y = -3x - 2$

3. $-0.45x - y = 1.35$
$-1.8x + y = -1.8$

4. $-0.4x + 0.8y = -16$
$1.2x + 0.4y = 1$

Use the matrix method to solve the linear systems.

5. $2x + y - 3z = -11$
$-x + 8y - z = 15$
$-4x - 5y - z = -5$

6. $-x - y + z = 3.2$
$y - 4z = -15.2$
$-2x + 5y + 2z = -2$

7. $w + 3x + 2y + z = 5$
$2w + x - y + z = 1$
$3w - 5x - y + 2z = -3$
$-w + x - 4y + z = 7$

8. Name two advantages and one disadvantage of the matrix method, compared to the graphical method.

Pre-AP Copymasters