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CHAPTER 6

Graph Systems of Two Equations and Three Equations

We already know that the graph of the solutions to a linear equation can be a vertical line, a horizontal line, or a line that lies at an angle to both the vertical and the horizontal axis. In this chapter we explore the solutions of *systems of linear equations*.

Among other things, we want to know what the solution graph for a system of equations looks like. To answer this question, we first need to consider what happens when we graph several equations in the same coordinate plane. We begin with scenarios involving just two equations in two variables.

EXAMPLE 1 Graph two equations in the same plane

For each of each of the following pairs of equations, graph the two equations together in one coordinate plane.

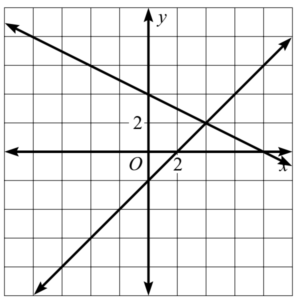
a. $x + 2y = 8$
$y = x - 2$

b. $y = -2x + 6$
$y = -2x - 3$

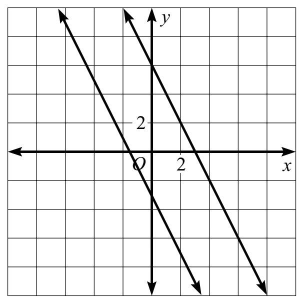
c. $2y - x = 4$
$-6y + 3x = -12$

Solution:

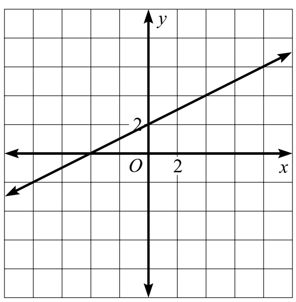
a.



b.



c.



■

The three graphs in Example 1 cover all the basic possibilities. Two linear equations may represent the same line (part c). But if the lines are distinct, they will either intersect (part a) or they will not (part b).

Now we can consider how the graphs of the equations in a system relate to the graph of the solution to the system as a whole.

KEY CONCEPT

Graphing the Solutions of a System of Equations

The graph of the solution or solutions of a system of equations consists of all points that lie on the line for every individual equation in the system.

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CHAPTER 6

Graph Systems of Two Equations and Three Equations *continued*

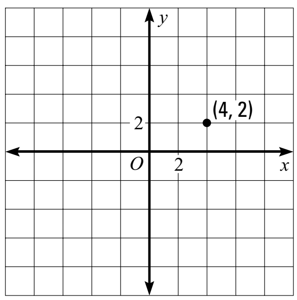
This principle comes from the definition of the solution of a system of equations: an ordered pair (x, y) is a solution of the system if it is a solution for every one of the individual equations.

EXAMPLE 2 Graph the solution of a two-equation linear system

Refer to Example 1, and graph the solution in each of the three cases.

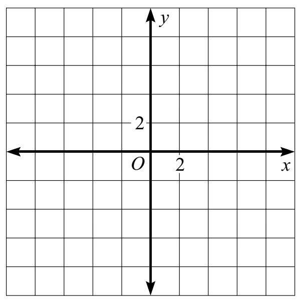
Solution:

a.



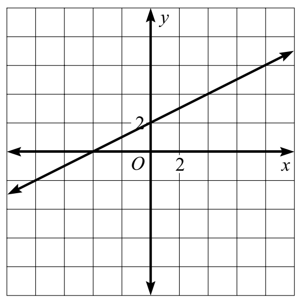
The point that lies on both lines is the point of intersection, (4, 2). That point is the graph of the solution of the sytem.

b.



No point lies on both lines, so the system has no solution. The graph of the solution is the empty graph.

c.



The two equations represent the same line. Every (x, y) pair on that line is a solution for both equations, and therefore is a solution for the system. ■

Part a in Examples 1 and 2 is the "typical" system. Systems like those in parts b and c are examined in more detail in Section 7.5.

What we have learned can be extended to systems of more than two equations. The solution of a single one-variable equation of the form $ax = b$ is a point (0 dimensions) on the number line (1 dimension). The solution of a single two-variable equation of the form $ax + by = c$ is a line (1 dimension) in the coordinate plane (2 dimensions). Similarly, the solution of a single three-variable equation of the form $ax + by + cz = d$ is a plane (2 dimensions) in the coordinate xyz-space (3 dimensions).

A similar pattern also applies to systems of equations as well. Just as the solution of a system of two linear equations in two variables is the intersection of two lines, the solution of a system of three linear equations in three variables is the intersection of three planes. As the number of dimensions grows, however, there are two problems. First, it gets harder to visualize and graph the solutions. And second, there are more and more ways for the solution graphs of the individual equations to intersect. With two equations in two variables, there are three possibilities. For three equations in three variables, there are four different possibilities for the intersection of the three solution planes: no intersection, a point, a line, or a plane.

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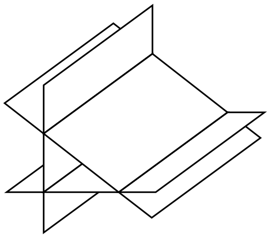
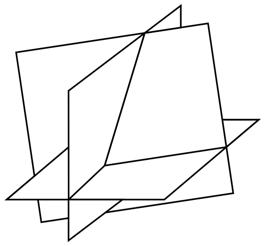
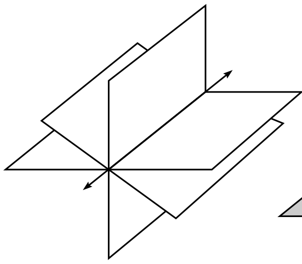
Graph Systems of Two Equations and Three Equations *continued*

Here are some illustrations of how each type of intersection may come about. Each illustration represents three planes in three-dimensional *xyz*-space. The axes are omitted for clarity.


No intersection


A point


A line


A plane

In the first case, each pair of planes has an intersection, but there is no point at which all three planes intersect. If these three planes are solution graphs for three linear equations, the system of equations has no solution.

In the second case, there is a single point at which all three planes intersect, and that point represents the solution of the system.

In the third and fourth cases, the system has more than one solution. In the third case, all solutions lie along a line running through the three-dimensional *xyz*-space, the line where all three planes intersect. And in the fourth case, all three equations have the same solution plane, and every point on that plane is a solution to the system.

Practice

Graph the solution of each system of equations. State whether the solution is a point, a line, or the empty graph.

1. $2x + y = 0$
$y = x + 3$

2. $x + y = 2$
$-3x - 3y = -6$

3. $3x + 2y = 2$
$4y = x - 10$

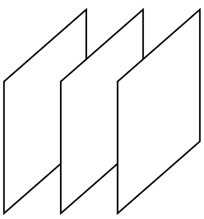
4. $-x + 4y = 8$
$4y = x - 12$

5. $3x + 8y = -20$
$16y + 40 = -6x$

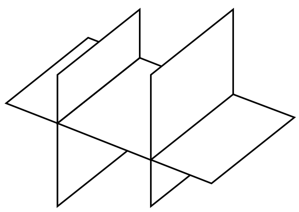
6. $x + 2y = -5$
$3y = 2x + 3$

Each of the following two figures depicts graphs of three linear equations in three variables. State whether the graph of the solution to the system of three equations is empty or is a point, a line, or a plane.

7.



8.



9. With three planes, it is possible for two planes to coincide but the third plane to be distinct. What two possibilities does this lead to for the intersection of all three planes? Draw both situations. If each plane represents the graph of a linear equation, what does each of the two outcomes imply about the solution of the system?