Name ______________________ Date ______________

LESSON 6.1

# Study Guide

*For use with the lesson "Use Combinations and the Binomial Theorem"*

**GOAL** **Use combinations and the binomial theorem.**

### Vocabulary

A **combination** is a selection of $r$ objects taken from a group of $n$ distinct objects where the order is not important.

If you arrange the values of ${}_nC_r$ in a triangular pattern in which each row corresponds to a value of $n$, you obtain **Pascal's triangle.**

The **Binomial Theorem** states that for any positive integer $n$, the binomial expansions of $(a + b)^n$ is

$$(a + b)^n = {}_nC_0a^nb^0 + {}_nC_1a^{n-1}b^1 + {}_nC_2a^{n-2}b^2 + \cdot \cdot \cdot + {}_nC_na^0b^n$$

where each term in the expansion of $(a + b)^n$ has the form ${}_nC_ra^{n-r}b^r$ where $r$ is an integer from 0 to $n$.

## EXAMPLE 1 Find combinations

**Committee Members** The board of directors of an organization has 7 members. In how many ways can the board choose a committee of 3 board members?

### Solution

Because order is not important, the number of ways to choose a committee of 3 board members from the board is:

$${}_7C_3 = \frac{7!}{4! \cdot 3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1} = 35$$

A committee of 3 board members can be chosen in 35 ways.

## EXAMPLE 2 Decide to multiply or add

**Sporting Events** Your school has scheduled three rugby games, four football games, and two soccer games.

**a.** In how many ways can you go to two rugby games and one other game?

**b.** In how many ways can you go to at least three of the four football games?

### Solution

**a.** You can choose 2 of 3 rugby games and 1 of the 6 other games. The number of possible choices is:

$${}_4C_2 \cdot {}_6C_1 = \frac{4!}{2! \cdot 2!} \cdot \frac{6!}{5!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{\cancel{2!} \cdot 2 \cdot 1} \cdot \frac{6 \cdot \cancel{5!}}{\cancel{5!}} = 6 \cdot 6 = 36$$

**b.** Of the four football games, you want to attend 3 games or 4 games, so you add the combinations. The number of combinations of games you can attend is:

$${}_4C_4 + {}_4C_3 = 1 + 4 = 5$$

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# Study Guide *continued*

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## Exercises for Examples 1 and 2

**Find the number of combinations.**

**1.** ${}_6C_4$ **2.** ${}_{10}C_7$ **3.** ${}_9C_3$ **4.** ${}_{13}C_2$

**5.** Rework Example 2 to find how many ways you can go to at least two of the four football games.

## EXAMPLE 3 Use Pascal's triangle

**Committee Members** Use Pascal's triangle to find the number of combinations of 3 committee members chosen from 8 available members.

**Solution**

To find ${}_8C_3$ write the 8th row of Pascal's triangle.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $n = 7$ (7th row) | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | |
| $n = 8$ (8th row) | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 |
| | ${}_8C_0$ | ${}_8C_1$ | ${}_8C_2$ | ${}_8C_3$ | ${}_8C_4$ | ${}_8C_5$ | ${}_8C_6$ | ${}_8C_7$ | ${}_8C_8$ |

The value of ${}_8C_3$ is the 4th number in the 8th row of Pascal's triangle, so ${}_8C_3 = 56$.

There are 56 combinations of 3 committee members.

## EXAMPLE 4 Expand a power of a binomial difference

**Use the binomial theorem to write the binomial expansion.**

$$\begin{aligned}(z - 3)^3 &= [z + (-3)]^3 \\ &= {}_3C_0z^3(-3)^0 + {}_3C_1z^2(-3)^1 + {}_3C_2z^1(-3)^2 + {}_3C_3z^0(-3)^3 \\ &= (1)(z^3)(1) + (3)(z^2)(-3) + (3)(z)(9) + (1)(1)(-27) \\ &= z^3 - 9z^2 + 27z - 27\end{aligned}$$

## EXAMPLE 5 Find a coefficient in an expansion

**Find the coefficient of $x^3$ in the expansion of $(4x + 3)^5$.**

Each term in the expansion has the form ${}_5C_r(4x)^{5-r}(3)^r$. The term containing $x^3$ occurs when $r = 2$:

${}_5C_2(4x)^3(3)^2 = (10)(64x^3)(9) = 5760x^3$

The coefficient of $x^3$ is 5760.

## Exercises for Examples 3, 4, and 5

**6.** Rework Example 3 choosing 4 committee members.

**7.** Use the binomial theorem to expand the expression $(x^3 + 2)^4$.

**8.** Find the coefficient of the $x^2$-term in Example 5.

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