6.1 Solve Linear Systems by Graphing

Before You graphed linear equations.

Now You will graph and solve systems of linear equations.

Why? So you can analyze craft fair sales, as in Ex. 33.

Key Vocabulary

- **system of linear equations**
- **solution of a system of linear equations**
- **consistent independent system**

A **system of linear equations,** or simply a *linear system*, consists of two or more linear equations in the same variables. An example is shown below.

$x + 2y = 7$ Equation 1

$3x - 2y = 5$ Equation 2

A **solution of a system of linear equations** in two variables is an ordered pair that satisfies each equation in the system.

One way to find the solution of a linear system is by graphing. If the lines intersect in a single point, then the coordinates of the point are the solution of the linear system. A solution found using graphical methods should be checked algebraically.

COMMON CORE

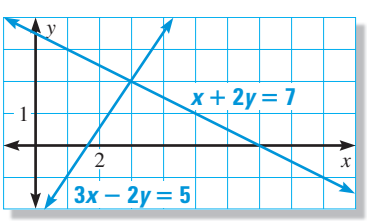
CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

EXAMPLE 1 Check the intersection point

Use the graph to solve the system. Then check your solution algebraically.

$x + 2y = 7$ Equation 1

$3x - 2y = 5$ Equation 2



Solution

The lines appear to intersect at the point (3, 2).

CHECK Substitute 3 for x and 2 for y in each equation.

$x + 2y = 7$

$3 + 2(2) \stackrel{?}{=} 7$

$7 = 7$ ✓

$3x - 2y = 5$

$3(3) - 2(2) \stackrel{?}{=} 5$

$5 = 5$ ✓

▶ Because the ordered pair (3, 2) is a solution of each equation, it is a solution of the system.

TYPES OF LINEAR SYSTEMS In Example 1, the linear system has exactly one solution. A linear system that has exactly one solution is called a **consistent independent system** because the lines are distinct (are independent) and intersect (are consistent). You will solve consistent independent systems in this chapter and you will also consider other types of systems.

KEY CONCEPT *For Your Notebook*

Solving a Linear System Using the Graph-and-Check Method

STEP 1 **Graph** both equations in the same coordinate plane. For ease of graphing, you may want to write each equation in slope-intercept form.

STEP 2 **Estimate** the coordinates of the point of intersection.

STEP 3 **Check** the coordinates algebraically by substituting into each equation of the original linear system.

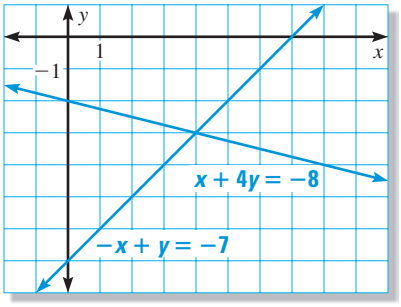
EXAMPLE 2 Use the graph-and-check method

Solve the linear system: $-x + y = -7$ Equation 1

$x + 4y = -8$ Equation 2

Solution

STEP 1 **Graph** both equations.



STEP 2 **Estimate** the point of intersection. The two lines appear to intersect at $(4, -3)$.

STEP 3 **Check** whether $(4, -3)$ is a solution by substituting 4 for x and -3 for y in each of the original equations.

Equation 1	Equation 2
$-x + y = -7$	$x + 4y = -8$
$-(4) + (-3) \stackrel{?}{=} -7$	$4 + 4(-3) \stackrel{?}{=} -8$
$-7 = -7$ ✓	$-8 = -8$ ✓

▶ Because $(4, -3)$ is a solution of each equation, it is a solution of the linear system.

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GUIDED PRACTICE for Examples 1 and 2

Solve the linear system by graphing. Check your solution.

1. $-5x + y = 0$
$5x + y = 10$

2. $-x + 2y = 3$
$2x + y = 4$

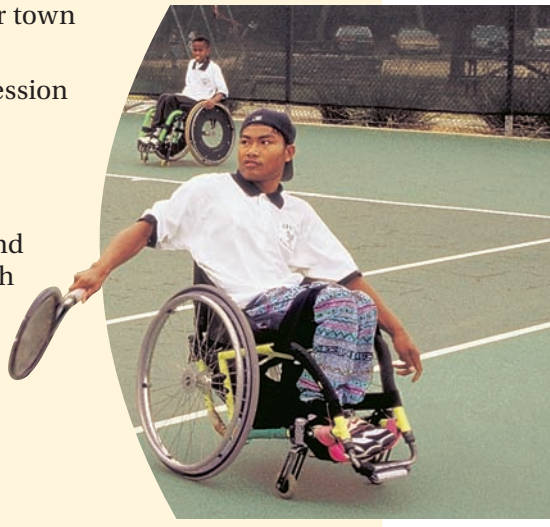
3. $x - y = 5$
$3x + y = 3$

EXAMPLE 3 Standardized Test Practice

The parks and recreation department in your town offers a season pass for $90.

- As a season pass holder, you pay $4 per session to use the town's tennis courts.
- Without the season pass, you pay $13 per session to use the tennis courts.

Which system of equations can be used to find the number x of sessions of tennis after which the total cost y with a season pass, including the cost of the pass, is the same as the total cost without a season pass?



ELIMINATE CHOICES
You can eliminate choice A because neither of the equations include the cost of a season pass.

(A) $y = 4x$, $y = 13x$ (B) $y = 4x$, $y = 90 + 13x$

(C) $y = 13x$, $y = 90 + 4x$ (D) $y = 90 + 4x$, $y = 90 + 13x$

Solution

Write a system of equations where y is the total cost (in dollars) for x sessions.

EQUATION 1

Total cost (dollars)	=	Cost per session (dollars/session)	·	Number of sessions (sessions)
y	=	13	·	x

EQUATION 2

Total cost (dollars)	=	Cost for season pass (dollars)	+	Cost per session (dollars/session)	·	Number of sessions (sessions)
y	=	90	+	4	·	x

▶ The correct answer is C. (A) (B) (C) (D)

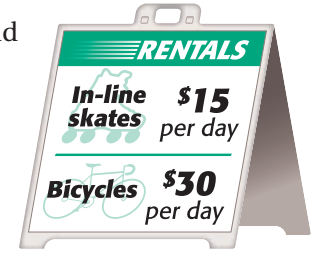
GUIDED PRACTICE for Example 3

4. Solve the linear system in Example 3 to find the number of sessions after which the total cost with a season pass, including the cost of the pass, is the same as the total cost without a season pass.

5. WHAT IF? In Example 3, suppose a season pass costs $135. After how many sessions is the total cost with a season pass, including the cost of the pass, the same as the total cost without a season pass?

EXAMPLE 4 Solve a multi-step problem

RENTAL BUSINESS A business rents in-line skates and bicycles. During one day, the business has a total of 25 rentals and collects $450 for the rentals. Find the number of pairs of skates rented and the number of bicycles rented.



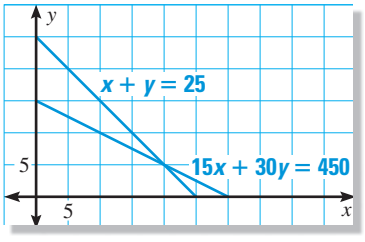
Solution

STEP 1 **Write** a linear system. Let x be the number of pairs of skates rented, and let y be the number of bicycles rented.

$x + y = 25$ Equation for number of rentals

$15x + 30y = 450$ Equation for money collected from rentals

STEP 2 **Graph** both equations.



STEP 3 **Estimate** the point of intersection. The two lines appear to intersect at (20, 5).

STEP 4 **Check** whether (20, 5) is a solution.

$20 + 5 \stackrel{?}{=} 25$

$25 = 25$ ✓

$15(20) + 30(5) \stackrel{?}{=} 450$

$450 = 450$ ✓

▶ The business rented 20 pairs of skates and 5 bicycles.

GUIDED PRACTICE for Example 4

6. **WHAT IF?** In Example 4, suppose the business has a total of 20 rentals and collects $420. Find the number of bicycles rented.

6.1 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 15 and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 7, 27, 28, 29, and 32

◆ = **MULTIPLE REPRESENTATIONS** Ex. 35

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A(n) ? of a system of linear equations in two variables is an ordered pair that satisfies each equation in the system.

2. ★ **WRITING** *Explain* how to use the graph-and-check method to solve a linear system of two equations in two variables.

CHECKING SOLUTIONS **Tell whether the ordered pair is a solution of the linear system.**

3. $(-3, 1)$;
$x + y = -2$
$x + 5y = 2$

4. $(5, 2)$;
$2x - 3y = 4$
$2x + 8y = 11$

5. $(-2, 1)$;
$6x + 5y = -7$
$x - 2y = 0$

EXAMPLE 1 for Exs. 6–11

6. ★ MULTIPLE CHOICE Which ordered pair is a solution of the linear system $x + y = -2$ and $7x - 4y = 8$?

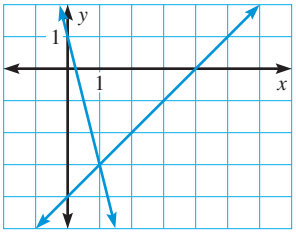
Ⓐ $(-2, 0)$ Ⓑ $(0, -2)$ Ⓒ $(2, 0)$ Ⓓ $(0, 2)$

7. ★ MULTIPLE CHOICE Which ordered pair is a solution of the linear system $2x + 3y = 12$ and $10x + 3y = -12$?

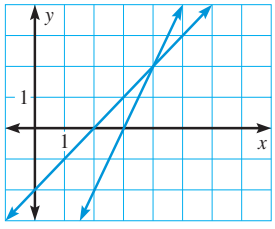
Ⓐ $(-3, 3)$ Ⓑ $(-3, 6)$ Ⓒ $(3, 3)$ Ⓓ $(3, 6)$

SOLVING SYSTEMS GRAPHICALLY **Use the graph to solve the linear system. Check your solution.**

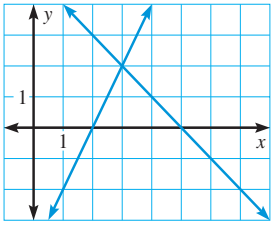
8. $x - y = 4$
$4x + y = 1$



9. $-x + y = -2$
$2x - y = 6$

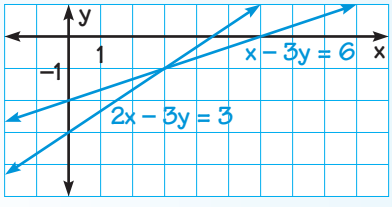


10. $x + y = 5$
$-2x + y = -4$



11. ERROR ANALYSIS *Describe* and correct the error in solving the linear system below.

$x - 3y = 6$ **Equation 1**
$2x - 3y = 3$ **Equation 2**



The solution is (3, –1).

EXAMPLE 2 for Exs. 12–26

GRAPH-AND-CHECK METHOD **Solve the linear system by graphing. Check your solution.**

12. $y = -x + 3$
$y = x + 1$

13. $y = -x + 4$
$y = 2x - 8$

14. $y = 2x + 2$
$y = 4x + 6$

15. $x - y = 2$
$x + y = -8$

16. $x + 2y = 1$
$-2x + y = -4$

17. $3x + y = 15$
$y = -15$

18. $2x - 3y = -1$
$5x + 2y = 26$

19. $6x + y = 37$
$4x + 2y = 18$

20. $7x + 5y = -3$
$-9x + y = -11$

21. $6x + 12y = -6$
$2x + 5y = 0$

22. $2x + y = 9$
$2x + 3y = 15$

23. $-5x + 3y = 3$
$4x + 3y = 30$

24. $\frac{3}{4}x + \frac{1}{4}y = \frac{13}{2}$
$x - \frac{3}{4}y = \frac{13}{2}$

25. $\frac{1}{5}x - \frac{2}{5}y = -\frac{8}{5}$
$-\frac{3}{4}x + y = 3$

26. $-1.6x - 3.2y = -24$
$2.6x + 2.6y = 26$

27. ★ OPEN-ENDED Find values for m and b so that the system $y = \frac{3}{5}x - 1$ and $y = mx + b$ has $(5, 2)$ as a solution.

28. ★ WRITING Solve the linear system shown by graphing. *Explain* why it is important to check your solution.

$y = 4x - 1.5$ **Equation 1**
$y = -2x + 1.5$ **Equation 2**

29. ★ **EXTENDED RESPONSE** Consider the equation $-\frac{1}{4}x + 6 = \frac{1}{2}x + 3$.

a. Solve the equation using algebra.

b. Solve the linear system below using a graph.

$y = -\frac{1}{4}x + 6$ **Equation 1**

$y = \frac{1}{2}x + 3$ **Equation 2**

c. How is the linear system in part (b) related to the original equation?

d. *Explain* how to use a graph to solve the equation $-\frac{2}{5}x + 5 = \frac{1}{5}x + 2$.

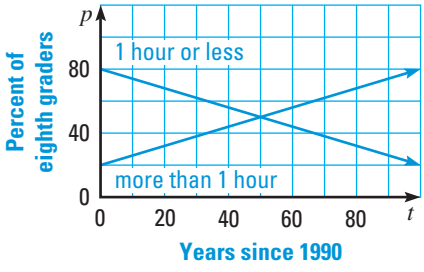
30. **CHALLENGE** The three lines given below form a triangle. Find the coordinates of the vertices of the triangle.

Line 1: $-3x + 2y = 1$ **Line 2:** $2x + y = 11$ **Line 3:** $x + 4y = 9$

PROBLEM SOLVING

EXAMPLES 3 and 4 for Exs. 31–33

31. **TELEVISION** The graph shows a projection, from 1990 on, of the percent of eighth graders who watch 1 hour or less of television on a weekday and the percent of eighth graders who watch more than 1 hour of television on a weekday. Use the graph to predict the year when the percent of eighth graders who watch 1 hour or less will equal the percent who watch more than 1 hour.



32. ★ **MULTIPLE CHOICE** A car dealership is offering interest-free car loans for one day only. During this day, a salesperson at the dealership sells two cars. One of his clients decides to pay off his \$17,424 car in 36 monthly payments of \$484. His other client decides to pay off his \$15,840 car in 48 monthly payments of \$330. Which system of equations can be used to determine the number x of months after which both clients will have the same loan balance y?

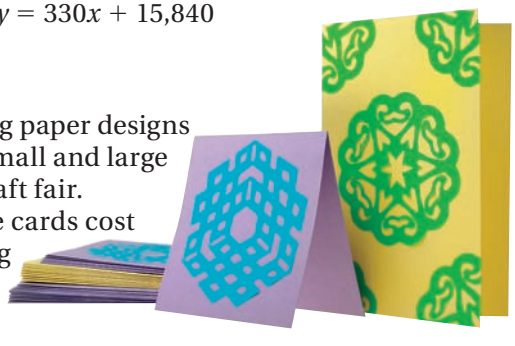
A $y = -484x$, $y = -330x$

B $y = -484x + 17{,}424$, $y = -330x + 15{,}840$

C $y = -484x + 15{,}840$, $y = -330x + 17{,}424$

D $y = 484x + 17{,}424$, $y = 330x + 15{,}840$

33. **CRAFTS** Kirigami is the Japanese art of making paper designs by folding and cutting paper. A student sells small and large greeting cards decorated with kirigami at a craft fair. The small cards cost \$3 per card, and the large cards cost \$5 per card. The student collects \$95 for selling a total of 25 cards. How many of each type of card did the student sell?



34. FITNESS You want to burn 225 calories while exercising at a gym. The number of calories that you burn per minute on different machines at the gym is shown below.

Stair machine	*Elliptical trainer*	*Stationary bike*
You burn 5 Cal/min. 	You burn 8 Cal/min. 	You burn 6 Cal/min. 

a. Suppose you have 40 minutes to exercise at the gym and you want to use the stair machine and stationary bike. How many minutes should you spend on each machine so that you burn 225 calories?

b. Suppose you have 30 minutes to exercise at the gym and you want to use the stair machine and the elliptical trainer. How many minutes should you spend on each machine so that you burn 225 calories?

35. MULTIPLE REPRESENTATIONS It costs $15 for a yearly membership to a movie club at a movie theater. A movie ticket costs $5 for club members and $8 for nonmembers.

a. Writing a System of Equations Write a system of equations that you can use to find the number x of movies viewed after which the total cost y for a club member, including the membership fee, is the same as the cost for a nonmember.

b. Making a Table Make a table of values that shows the total cost for a club member and a nonmember after paying to see 1, 2, 3, 4, 5, and 6 movies.

c. Drawing a Graph Use the table to graph the system of equations. Under what circumstances does it make sense to become a movie club member? *Explain* your answer by using the graph.

36. CHALLENGE With a minimum purchase of $25, you can open a credit account with a clothing store. The store is offering either $25 or 20% off of your purchase if you open a credit account. You decide to open a credit account. Should you choose $25 or 20% off of your purchase? *Explain*.

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Keystrokes

Solving Linear Systems by Graphing

MATHEMATICAL PRACTICES
Use appropriate tools strategically.

QUESTION How can you use a graphing calculator to solve a linear system?

EXAMPLE Solve a linear system

Solve the linear system using a graphing calculator.

$5x + 2y = 6$ Equation 1
$x - 3y = -5$ Equation 2

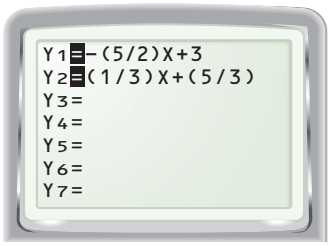
STEP 1 Rewrite equations

Solve each equation for y.

Equation 1	Equation 2
$5x + 2y = 6$	$x - 3y = -5$
$2y = -5x + 6$	$-3y = -x - 5$
$y = -\frac{5}{2}x + 3$	$y = \frac{1}{3}x + \frac{5}{3}$

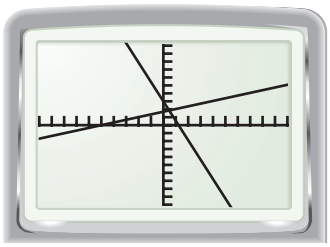
STEP 2 Enter equations

Press Y= and enter the equations.



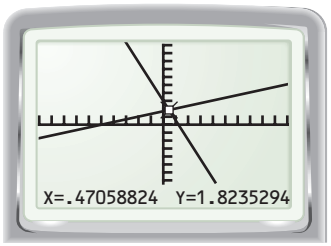
STEP 3 Display graph

Graph the equations using a standard viewing window.



STEP 4 Find point of intersection

Use the *intersect* feature to find the point where the graphs intersect.



The solution is about (0.47, 1.8).

PRACTICE

Solve the linear system using a graphing calculator.

1. $y = x + 4$
$y = -3x - 2$

2. $5x + y = -4$
$x - y = -2$

3. $-0.45x - y = 1.35$
$-1.8x + y = -1.8$

4. $-0.4x + 0.8y = -16$
$1.2x + 0.4y = 1$