# Use a Simulation to Test an Assumption 

Use appropriate tools strategically.
MATERIALS•coins •graphing calculator

## QUESTION How do you determine whether a coin is fair?

How do you know if a coin is "fair"? That is, when you flip it, how do you know a coin is equally likely to land heads or tails?

You can flip an actual coin many times to help you decide whether you think it's fair. But what kinds of seemingly "unusual" outcomes might occur even with a fair coin? How do you know what you can expect?

In this activity, you will perform physical experiments with a coin and also use simulation with a graphing calculator to model flipping a coin. The simulation lets you quickly repeat an event with two equally likely outcomes to compare with results that you might get from an actual coin.

## Explore 1 Perform an experiment

## A friend gives you a coin. You flip it 4 times, and you get tails all 4 times.

 Would you feel confident in concluding that the coin is not fair?
## STEP 1 Flip coins

Working in pairs, flip a coin 4 times. Repeat this experiment a total of 10 times, so that you have 10 trials of 4 flips. Record your results in a table like the one shown. Count the number of trials that fall into each of these categories: 4 heads, 3 heads and 1 tails, 2 heads and 2 tails, 1 heads and 3 tails, 4 tails.

|  | Flip 1 | Flip 2 | Flip 3 | Flip 4 |
| :---: | :---: | :---: | :---: | :---: |
| Trial 1 | ? | ? | $?$ | $?$ |
| Trial 2 | $?$ | $?$ | $?$ | $?$ |
|  |  |  |  |  |

## STEP 2 Display results

Collect all of the data generated by the pairs of students in your class. Combine the outcomes for all of the trials, and then display the class data using a bar graph set up like the one at the right.

## STEP 3 Analyze results

What outcome or outcomes occurred the most frequently? How many times did 4 heads or 4 tails occur? Would you be reasonably
 certain concluding that the coin your friend gave you is not fair?
Explain.

## EXPLORE 2 Perform a simulation of an experiment

## A friend gives you a coin. You flip it 7 times, and you get tails all 7 times. Would you feel confident in concluding that the coin is not fair?

## STEP 1 Generate data

For the simulation, let 0 represent flipping heads and let 1 represent flipping tails. On a graphing calculator, press the
randInt (0, 1, 7)
\{0 1100010$\}$

MATH key, select the "PRB" menu, and choose "randInt(."
Perform the keystrokes for "randInt ( $0,1,7$ )" and press ENTER.
The calculator will produce a list of seven digits (0 or 1) at random, as shown, where the outcomes 0 and 1 are equally likely to occur.

Press ENTER 20 times to obtain 20 lists of 7 "coin flips." Keep a tally to record the number of these 20 trials whose results fall into each of these categories: 7 heads, 6 heads and 1 tail, 5 heads and 2 tails, ..., 1 heads and 6 tails, 7 tails.

## STEP 2 Display results

Collect all of the data generated by the pairs in your class. Combine the outcomes for all of the trials and display the class data using a bar graph similar to the one used in Explore 1. It will have 8 bars.

## STEP 3 Analyze results

What outcome or outcomes occurred the most? How many times did 7 heads or 7 tails occur? Would you be reasonably certain concluding that the coin your friend gave you is not fair? Explain.

## Draw Conclusions Use your observations to complete these exercises

1. In Explore 1, find the theoretical probability of flipping 4 tails in a row. Does this affect your confidence in the coin's fairness or unfairness? Explain.
2. In Explore 2, find the theoretical probability of flipping 7 tails in a row. Does this affect your confidence in the coin's fairness or unfairness? Explain.
3. A simulation of flipping a coin 5 times in a row is performed 200 times. The results of the simulation are shown in the table below.

|  | 5 heads <br> $\mathbf{0}$ tails | $\mathbf{4}$ heads <br> $\mathbf{1}$ tails | 3 heads <br> $\mathbf{2}$ tails | $\mathbf{2}$ heads <br> $\mathbf{3}$ tails | $\mathbf{1}$ heads <br> $\mathbf{4}$ tails | $\mathbf{0}$ heads <br> $\mathbf{5}$ tails |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Outcomes | 1 | 5 | 27 | 61 | 72 | 34 |

a. Make a bar graph of the data.
b. Do you think that the simulation represents a coin that is fair or not fair? Explain.

