

# 6.2 Solve Linear Systems by Substitution



**Before**

You solved systems of linear equations by graphing.

**Now**

You will solve systems of linear equations by substitution.

**Why?**

So you can find tubing costs, as in Ex. 32.

## Key Vocabulary

- system of linear equations

COMMON CORE

**CC.9-12.A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

## KEY CONCEPT

*For Your Notebook*

### Solving a Linear System Using the Substitution Method

- STEP 1** **Solve** one of the equations for one of its variables. When possible, solve for a variable that has a coefficient of 1 or  $-1$ .
- STEP 2** **Substitute** the expression from Step 1 into the other equation and solve for the other variable.
- STEP 3** **Substitute** the value from Step 2 into the revised equation from Step 1 and solve.

## EXAMPLE 1 Use the substitution method

Solve the linear system:  $y = 3x + 2$  Equation 1  
 $x + 2y = 11$  Equation 2

### Solution

**STEP 1** **Solve** for  $y$ . Equation 1 is already solved for  $y$ .

**STEP 2** **Substitute**  $3x + 2$  for  $y$  in Equation 2 and solve for  $x$ .

$$x + 2y = 11 \quad \text{Write Equation 2.}$$

$$x + 2(3x + 2) = 11 \quad \text{Substitute } 3x + 2 \text{ for } y.$$

$$7x + 4 = 11 \quad \text{Simplify.}$$

$$7x = 7 \quad \text{Subtract 4 from each side.}$$

$$x = 1 \quad \text{Divide each side by 7.}$$

**STEP 3** **Substitute** 1 for  $x$  in the original Equation 1 to find the value of  $y$ .

$$y = 3x + 2 = 3(1) + 2 = 3 + 2 = 5$$

► The solution is  $(1, 5)$ .

**CHECK** Substitute 1 for  $x$  and 5 for  $y$  in each of the original equations.

$$y = 3x + 2$$

$$5 \stackrel{?}{=} 3(1) + 2$$

$$5 = 5 \quad \checkmark$$

$$x + 2y = 11$$

$$1 + 2(5) \stackrel{?}{=} 11$$

$$11 = 11 \quad \checkmark$$

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**EXAMPLE 2** Use the substitution method

Solve the linear system:  $x - 2y = -6$  Equation 1  
 $4x + 6y = 4$  Equation 2

**CHOOSE AN EQUATION**

Equation 1 was chosen in Step 1 because  $x$  has a coefficient of 1. So, only one step is needed to solve Equation 1 for  $x$ .

**Solution**

► **STEP 1** Solve Equation 1 for  $x$ .

$$x - 2y = -6 \quad \text{Write original Equation 1.}$$

$$x = 2y - 6 \quad \text{Revised Equation 1}$$

**STEP 2** Substitute  $2y - 6$  for  $x$  in Equation 2 and solve for  $y$ .

$$4x + 6y = 4 \quad \text{Write Equation 2.}$$

$$4(2y - 6) + 6y = 4 \quad \text{Substitute } 2y - 6 \text{ for } x.$$

$$8y - 24 + 6y = 4 \quad \text{Distributive property}$$

$$14y - 24 = 4 \quad \text{Simplify.}$$

$$14y = 28 \quad \text{Add 24 to each side.}$$

$$y = 2 \quad \text{Divide each side by 14.}$$

**STEP 3** Substitute 2 for  $y$  in the revised Equation 1 to find the value of  $x$ .

$$x = 2y - 6 \quad \text{Revised Equation 1}$$

$$x = 2(2) - 6 \quad \text{Substitute 2 for } y.$$

$$x = -2 \quad \text{Simplify.}$$

► The solution is  $(-2, 2)$ .

**CHECK** Substitute  $-2$  for  $x$  and  $2$  for  $y$  in each of the original equations.

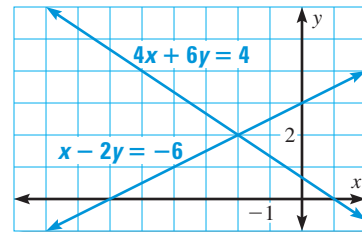
**Equation 1**

$$\begin{aligned} x - 2y &= -6 \\ -2 - 2(2) &\stackrel{?}{=} -6 \\ -6 &= -6 \quad \checkmark \end{aligned}$$

**Equation 2**

$$\begin{aligned} 4x + 6y &= 4 \\ 4(-2) + 6(2) &\stackrel{?}{=} 4 \\ 4 &= 4 \quad \checkmark \end{aligned}$$

**CHECK REASONABLENESS** When solving a linear system using the substitution method, you can use a graph to check the reasonableness of your solution. For example, the graph at the right verifies that  $(-2, 2)$  is a solution of the linear system in Example 2.

**GUIDED PRACTICE** for Examples 1 and 2

Solve the linear system using the substitution method.

1.  $y = 2x + 5$   
 $3x + y = 10$

2.  $x - y = 3$   
 $x + 2y = -6$

3.  $3x + y = -7$   
 $-2x + 4y = 0$

**EXAMPLE 3** Solve a multi-step problem**ANOTHER WAY**

For an alternative method for solving the problem in Example 3, see the **Problem Solving Workshop**.

**WEBSITES** Many businesses pay website hosting companies to store and maintain the computer files that make up their websites. Internet service providers also offer website hosting. The costs for website hosting offered by a website hosting company and an Internet service provider are shown in the table. Find the number of months after which the total cost for website hosting will be the same for both companies.

Company	Set-up fee (dollars)	Cost per month (dollars)
Internet service provider	10	21.95
Website hosting company	None	22.45

**Solution**

**STEP 1** Write a system of equations. Let  $y$  be the total cost after  $x$  months.

**Equation 1: Internet service provider**

Total cost	=	Set-up fee	+	Cost per month	·	Number of months
$y$	=	10	+	21.95	·	$x$

**Equation 2: Website hosting company**

Total cost	=	Cost per month	·	Number of months
$y$	=	22.45	·	$x$

The system of equations is:  $y = 10 + 21.95x$  **Equation 1**  
 $y = 22.45x$  **Equation 2**

**STEP 2** Substitute  $22.45x$  for  $y$  in Equation 1 and solve for  $x$ .

$$y = 10 + 21.95x \quad \text{Write Equation 1.}$$

$$22.45x = 10 + 21.95x \quad \text{Substitute } 22.45x \text{ for } y.$$

$$0.5x = 10 \quad \text{Subtract } 21.95x \text{ from each side.}$$

$$x = 20 \quad \text{Divide each side by 0.5.}$$

► The total cost will be the same for both companies after 20 months.

**GUIDED PRACTICE** for Example 3

- In Example 3, what is the total cost for website hosting for each company after 20 months?
- WHAT IF?** In Example 3, suppose the Internet service provider offers \$5 off the set-up fee. After how many months will the total cost for website hosting be the same for both companies?

**EXAMPLE 4** Solve a mixture problem

**ANTIFREEZE** For extremely cold temperatures, an automobile manufacturer recommends that a 70% antifreeze and 30% water mix be used in the cooling system of a car. How many quarts of pure (100%) antifreeze and a 50% antifreeze and 50% water mix should be combined to make 11 quarts of a 70% antifreeze and 30% water mix?


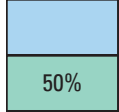
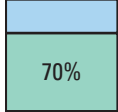
**Solution**

**STEP 1** Write an equation for the total number of quarts and an equation for the number of quarts of antifreeze. Let  $x$  be the number of quarts of 100% antifreeze, and let  $y$  be the number of quarts of a 50% antifreeze and 50% water mix.

**Equation 1: Total number of quarts**

$$x + y = 11$$

**Equation 2: Number of quarts of antifreeze**

$x$ quarts of 100% antifreeze	$y$ quarts of 50%–50% mix	11 quarts of 70%–30% mix
		
$1 \cdot x$	$+ \quad 0.5 \cdot y$	$= \quad 0.7(11)$
$x + 0.5y = 7.7$		

The system of equations is:  $x + y = 11$  Equation 1  
 $x + 0.5y = 7.7$  Equation 2

**STEP 2** Solve Equation 1 for  $x$ .

$$x + y = 11 \quad \text{Write Equation 1.}$$

$$x = 11 - y \quad \text{Revised Equation 1}$$

**STEP 3** Substitute  $11 - y$  for  $x$  in Equation 2 and solve for  $y$ .

$$x + 0.5y = 7.7 \quad \text{Write Equation 2.}$$

$$(11 - y) + 0.5y = 7.7 \quad \text{Substitute } 11 - y \text{ for } x.$$

$$y = 6.6 \quad \text{Solve for } y.$$

**STEP 4** Substitute 6.6 for  $y$  in the revised Equation 1 to find the value of  $x$ .

$$x = 11 - y = 11 - 6.6 = 4.4$$

► Mix 4.4 quarts of 100% antifreeze and 6.6 quarts of a 50% antifreeze and 50% water mix to get 11 quarts of a 70% antifreeze and 30% water mix.

**DRAW A DIAGRAM**

Each bar shows the liquid in each mix. The green portion shows the percent of the mix that is antifreeze.

**GUIDED PRACTICE** for Example 4

6. **WHAT IF?** How many quarts of 100% antifreeze and a 50% antifreeze and 50% water mix should be combined to make 16 quarts of a 70% antifreeze and 30% water mix?

# 6.2 EXERCISES

## HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS**  
Exs. 13 and 33

★ = **STANDARDIZED TEST PRACTICE**  
Exs. 2, 18, 29, 33, and 37

## SKILL PRACTICE

1. **VOCABULARY** Give an example of a system of linear equations.

2. ★ **WRITING** If you are solving the linear system shown using the substitution method, which equation would you solve for which variable? Explain.

$$2x - 3y = 24 \quad \text{Equation 1}$$

$$2x + y = 8 \quad \text{Equation 2}$$

**EXAMPLE 1**  
for Exs. 3–8

**SOLVING LINEAR SYSTEMS** Solve the linear system using substitution.

3.  $x = 17 - 4y$   
 $y = x - 2$

4.  $y = 2x - 1$   
 $2x + y = 3$

5.  $x = y + 3$   
 $2x - y = 5$

6.  $4x - 7y = 10$   
 $y = x - 7$

7.  $x = 16 - 4y$   
 $3x + 4y = 8$

8.  $-5x + 3y = 51$   
 $y = 10x - 8$

9.  $2x = 12$   
 $x - 5y = -29$

10.  $2x - y = 23$   
 $x - 9 = -1$

11.  $x + y = 0$   
 $x - 2y = 6$

12.  $2x + y = 9$   
 $4x - y = -15$

13.  $5x + 2y = 9$   
 $x + y = -3$

14.  $5x + 4y = 32$   
 $9x - y = 33$

15.  $11x - 7y = -14$   
 $x - 2y = -4$

16.  $20x - 30y = -50$   
 $x + 2y = 1$

17.  $6x + y = 4$   
 $x - 4y = 19$

18. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the linear system  $4x - y = 17$  and  $-9x + 8y = 2$ ?

(A) (6, 7)

(B) (7, 6)

(C) (7, 11)

(D) (11, 7)

19. **ERROR ANALYSIS** Describe and correct the error in solving the linear system  $4x + 2y = 6$  and  $3x + y = 9$ .

Step 1

$$3x + y = 9$$

$$y = 9 - 3x$$

Step 2

$$4x + 2(9 - 3x) = 6$$

$$4x + 18 - 6x = 6$$

$$-2x = -12$$

$$x = 6$$

Step 3

$$y = 9 - 3x$$

$$6 = 9 - 3x$$

$$-3 = -3x$$

$$1 = x$$

The solution is (6, 1).



**SOLVING LINEAR SYSTEMS** Solve the linear system using substitution.

20.  $4.5x + 1.5y = 24$   
 $x - y = 4$

21.  $35x + y = 20$   
 $1.5x - 0.1y = 18$

22.  $3x - 2y = 8$   
 $0.5x + y = 17$

23.  $0.5x + 0.6y = 5.7$   
 $2x - y = -1$

24.  $x - 9 = 0.5y$   
 $2.2x - 3.1y = -0.2$

25.  $0.2x + y = -1.8$   
 $1.8y + 5.5x = 27.6$

26.  $\frac{1}{2}x + \frac{1}{4}y = 5$   
 $x - \frac{1}{2}y = 1$

27.  $x + \frac{1}{3}y = -2$   
 $-8x - \frac{2}{3}y = 4$

28.  $\frac{3}{8}x + \frac{3}{4}y = 12$   
 $\frac{2}{3}x + \frac{1}{2}y = 13$

29. ★ **WRITING** Suppose you solve a linear system using substitution. Explain how you can use a graph to check your solution.

30. **CHALLENGE** Find values of  $a$  and  $b$  so that the linear system shown has a solution of  $(-9, 4)$ .
- $$ax + by = -16 \quad \text{Equation 1}$$
- $$ax - by = -56 \quad \text{Equation 2}$$

## PROBLEM SOLVING

**EXAMPLE 3**  
for Exs. 31–33

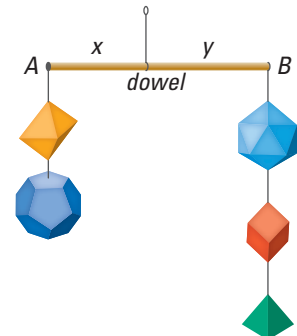
31. **FUNDRAISING** During a football game, the parents of the football players sell pretzels and popcorn to raise money for new uniforms. They charge \$2.50 for a bag of popcorn and \$2 for a pretzel. The parents collect \$336 in sales during the game. They sell twice as many bags of popcorn as pretzels. How many bags of popcorn do they sell? How many pretzels do they sell?

32. **TUBING COSTS** A group of friends takes a day-long tubing trip down a river. The company that offers the tubing trip charges \$15 to rent a tube for a person to use and \$7.50 to rent a “cooler” tube, which is used to carry food and water in a cooler. The friends spend \$360 to rent a total of 26 tubes. How many of each type of tube do they rent?

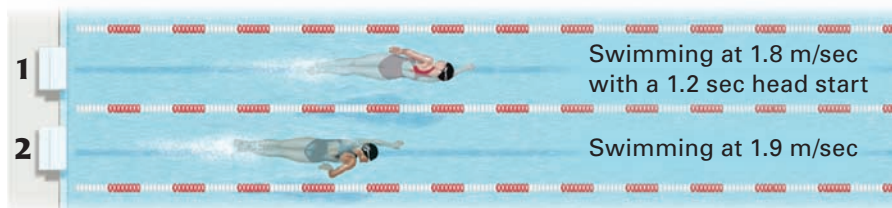
33. ★ **SHORT RESPONSE** In the mobile shown, objects are attached to each end of a dowel. For the dowel to balance, the following must be true:

$$x \cdot \begin{array}{c} \text{Weight hanging} \\ \text{from point A} \end{array} = y \cdot \begin{array}{c} \text{Weight hanging} \\ \text{from point B} \end{array}$$

The weight of the objects hanging from point A is 1.5 pounds, and the weight of the objects hanging from point B is 1.2 pounds. The length of the dowel is 9 inches. How far from point A should the string be placed? *Explain.*




34. **MULTI-STEP PROBLEM** Two swimming teams are competing in a 400 meter medley relay. During the last leg of the race, the swimmer in lane 1 has a 1.2 second head start on the swimmer in lane 2, as shown.



- Let  $t$  be the time since the swimmer in lane 2 started the last leg. After how many seconds into the leg will the swimmer in lane 2 catch up to the swimmer in lane 1?
- Does the swimmer in lane 2 catch up to the swimmer in lane 1 before the race ends? *Explain.*

**EXAMPLE 4**  
for Ex. 35

35. **CHEMISTRY** In your chemistry lab, you have a bottle of 1% hydrochloric acid solution and a bottle of 5% hydrochloric acid solution. You need 100 milliliters of a 3% hydrochloric acid solution for an experiment. How many milliliters of each solution do you need to mix together?
36. **MONEY** Laura has \$4.50 in dimes and quarters. She has 3 more dimes than quarters. How many quarters does she have?
37. **★ SHORT RESPONSE** A gazelle can run 73 feet per second for several minutes. A cheetah can run 88 feet per second, but it can sustain this speed for only 20 seconds. A gazelle is 350 feet from a cheetah when both animals start running. Can the gazelle stay ahead of the cheetah? *Explain.*
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38. **CHALLENGE** A gardener needs 6 bushels of a potting medium of 40% peat moss and 60% vermiculite. He decides to add 100% vermiculite to his current potting medium that is 50% peat moss and 50% vermiculite. The gardener has 5 bushels of the 50% peat moss and 50% vermiculite mix. Does he have enough of the 50% peat moss and 50% vermiculite mix to make 6 bushels of the 40% peat moss and 60% vermiculite mix? *Explain.*

## QUIZ

Solve the linear system by graphing. Check your solution.

1.  $x + y = -2$   
 $-x + y = 6$

2.  $x - y = 0$   
 $5x + 2y = -7$

3.  $x - 2y = 12$   
 $-3x + y = -1$

Solve the linear system using substitution.

4.  $y = x - 4$   
 $-2x + y = 18$

5.  $y = 4 - 3x$   
 $5x - y = 22$

6.  $x = y + 9$   
 $5x - 3y = 7$

7.  $2y + x = -4$   
 $y - x = -5$

8.  $5x - 4y = 27$   
 $-2x + y = 3$

9.  $3x - 5y = 13$   
 $x + 4y = 10$





### Another Way to Solve Example 3



Make sense of problems and persevere in solving them.

### PROBLEM

**WEBSITES** Many businesses pay website hosting companies to store and maintain the computer files that make up their websites. Internet service providers also offer website hosting. The costs for website hosting offered by a website hosting company and an Internet service provider are shown in the table. Find the number of months after which the total cost for website hosting will be the same for both companies.

Company	Set-up fee	Cost per month
Internet service provider	\$10	\$21.95
Website hosting company	None	\$22.45

### METHOD

**Making a Table** An alternative approach is to make a table.

**STEP 1** **Make** a table for the total cost of website hosting for both companies.

Include the set-up fee in the cost for the first month.

**STEP 2** **Look** for the month in which the total cost of the service from the Internet service provider and the website hosting company is the same. This happens after 20 months.

Months	Internet service provider	Website hosting company
1	\$31.95	\$22.45
2	\$53.90	\$44.90
3	\$75.85	\$67.35
⋮	⋮	⋮
19	\$427.05	\$426.55
20	\$449.00	\$449.00
21	\$470.95	\$471.45

### PRACTICE

- TAXI** A taxi company charges \$2.80 for the first mile and \$1.60 for each additional mile. Another taxi company charges \$3.20 for the first mile and \$1.50 for each additional mile. After how many miles will each taxi cost the same? Use a table to solve the problem.
- SCHOOL PLAY** An adult ticket to a school play costs \$5 and a student ticket costs \$3. A total of \$460 was collected from the sale of 120 tickets. How many student tickets were purchased? Solve the problem using algebra. Then use a table to check your answer.



# Linear Systems and Elimination

**MATERIALS** • algebra tiles



Use appropriate tools strategically.

**QUESTION** How can you solve a linear system using algebra tiles?

You can use the following algebra tiles to model equations.

1-tiles



x-tiles



y-tiles



**EXPLORE** Solve a linear system using algebra tiles.

Solve the linear system:

$$3x - y = 5$$

Equation 1

$$x + y = 3$$

Equation 2

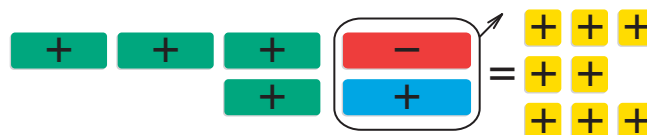
**STEP 1** Model equations

Model each equation using algebra tiles. Arrange the algebra tiles so that one equation is directly below the other equation.



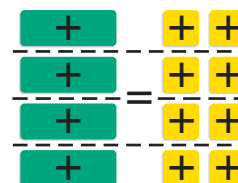
**STEP 2** Add equations

Combine the two equations to form one equation. Notice that the new equation has one positive y-tile and one negative y-tile. The y-tiles can be removed because the pair of y-tiles has a value of 0.



**STEP 3** Solve for x

Divide the remaining tiles into four equal groups. Each x-tile is equal to two 1-tiles. So,  $x = 2$ .



**STEP 4** Solve for y

To find the value of y, use the model for Equation 2. Because  $x = 2$ , you can replace the x-tile with two 1-tiles. Solve the new equation for y. So  $y = 1$ , and the solution of the system is (2, 1).



**DRAW CONCLUSIONS** Use your observations to complete these exercises

Use algebra tiles to model and solve the linear system.

1.  $x + 3y = 8$   
 $4x - 3y = 2$

2.  $2x + y = 5$   
 $-2x + 3y = 7$

3.  $5x - 2y = -2$   
 $x + 2y = 14$

4.  $x + 2y = 3$   
 $-x + 3y = 2$

5. **REASONING** Is it possible to solve the linear system  $3x - 2y = 6$  and  $2x + y = 11$  using the steps shown above? Explain your reasoning.