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CHAPTER 6

More on Probability Distributions

A **probability distribution** is a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution is equal to 1.

EXAMPLE 1

Find a uniform distribution

A bag contains five toys. As a reward, a child is allowed to reach in the bag and blindly select a toy. Find the probability distribution of this situation.

Solution:

There is a $\frac{1}{5} = 0.2$ probability of selecting each toy from the bag. The table shows the probability distribution. The function Toy(X) gives the probability of selecting Toy(X), where X ranges from 1 to 5.

Toy(X)	Toy(1)	Toy(2)	Toy(3)	Toy(4)	Toy(5)
Probability	0.2	0.2	0.2	0.2	0.2

Notice that the sum of all the probabilities is equal to 1. ■

A **relative-frequency histogram** is a graph that shows the probability distribution. If the width of each bar in a relative–frequency histogram is assumed to be equal to 1, then the area of each bar, and the area of the entire histogram, is easily calculated.

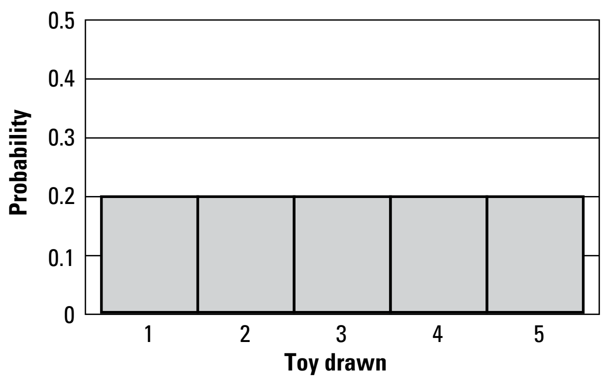
EXAMPLE 2

Find the area of a uniform relative-frequency distribution

Find the area of the probability distribution from Example 1.

Solution:

We can construct a relative–frequency histogram as shown below.



The area of each bar of this histogram is 0.2 (base times height), and because there are a total of 5 bars, the total area of the histogram is equal to 1. ■

In the next few examples, we will calculate the areas of **binomial distributions**. Recall that binomial distributions are constructed from binomial experiments.

EXAMPLE 3

Find a probability distribution and its area

Alicia plays on the soccer team. Based on the team's record, it has been determined that there is a 0.6 probability of winning a game. What is the probability distribution for the next two games?

Solution:

Losing both games:

$P(0 \text{ out of } 2) = \frac{2!}{0!(2-0)!} \cdot (0.6)^0 (0.4)^2 = 0.16$

Winning one of the next two games:

$P(1 \text{ out of } 2) = \frac{2!}{1!(2-1)!} \cdot (0.6)^1 (0.4)^1 = 0.48$

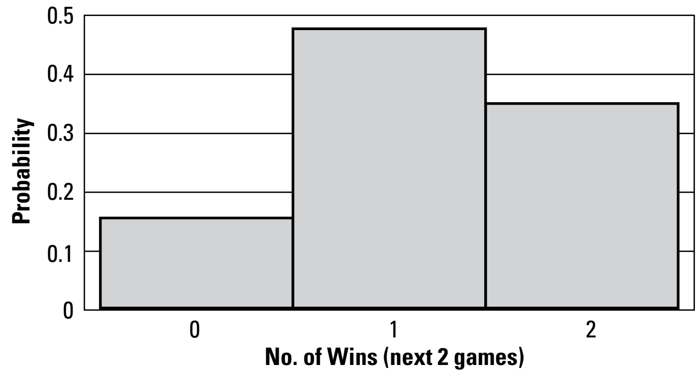
Winning both games:

$P(2 \text{ out of } 2) = \frac{2!}{2!(2-2)!} \cdot (0.6)^2 (0.4)^0 = 0.36$

The probability distribution is shown in the table below.

Number of Wins (X)	0	1	2
Probability	0.16	0.48	0.36

Construct a histogram of the binomial distribution.



Using a bar width of 1, we can calculate the total area of the distribution.

Total area = 1(0.16) + 1(0.48) + 1(0.36) = 1

The total area of the distribution is equal to 1. ■

EXAMPLE 4 Find a probability distribution and its area

Let X be a random variable that represents the number of tails in a three-coin toss. Find the probability distribution and its area.

Solution:

No tails:

$P(0 \text{ out of } 3) = \frac{3!}{0!(3-0)!} \cdot (0.5)^0 (0.5)^3 = 0.125$

One tail:

$P(1 \text{ out of } 3) = \frac{3!}{1!(3-1)!} \cdot (0.5)^1 (0.5)^2 = 0.375$

Two tails:

$P(2 \text{ out of } 3) = \frac{3!}{2!(3-2)!} \cdot (0.5)^2 (0.5)^1 = 0.375$

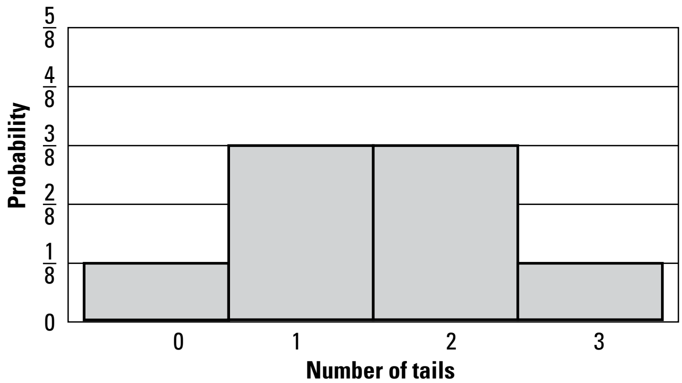
Three tails:

$P(3 \text{ out of } 3) = \frac{3!}{3!(3-3)!} \cdot (0.5)^3 (0.5)^0 = 0.125$

This distribution is shown in the table below.

Number of Tails (X)	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Construct a histogram of the binomial distribution.



If we take the width of each bar in the distribution to be 1, we can calculate the sum of the areas of the bars as follows:

Total area $= 1 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 1 \cdot \frac{3}{8} + 1 \cdot \frac{1}{8} = 1$

The total area of the binomial distribution is equal to 1. ■

We have now seen several examples of binomial probability distributions. On each example, the sum of the probabilities and the area of the related histogram is 1. We can extend this idea to any size binomial experiment.

KEY CONCEPT

For a binomial experiment of n trials,

$$\sum_{k=0}^{n} P(k \text{ successes}) = 1$$

EXAMPLE 5 **Calculate probabilities**

Consider the binomial distribution from Example 4. What is the probability of obtaining fewer than three 3 tails in a three-coin toss?

Solution:

This can be written as follows:

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$$
$$= \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$$
$$= \frac{7}{8}$$

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CHAPTER 6

More on Probability Distributions *continued*

Notice that if we subtract this result from 1 (the area of the histogram), we obtain exactly the same result as $P(X = 3)$.

$P(X = 3) = 1 - P(X < 3) = 1 - \frac{7}{8} = \frac{1}{8}$ ■

Practice

1. Kenneth is one of the leading players on the basketball team. During a game, he typically makes 3 out of every 5 basket attempts.

a. The table below shows the probability distribution for the next 4 basket attempts. Complete the table.

Number of Baskets (X)	0	1	2	3	4
Probability					

b. Construct a histogram of the binomial distribution.

c. Verify that the total area of the histogram is equal to 1.

d. Compute $P(X > 3)$.

e. Use your answer from part (d) to compute $P(X \leq 3)$.

2. Carla tosses a coin five times in a row.

a. The table below shows the probability distribution for the number of tails. Complete the table.

Number of Tails (X)	0	1	2	3	4	5
Probability						

b. Construct a histogram of the binomial distribution.

c. Verify that the total area of the histogram is equal to 1.

d. Compute $P(X < 2)$.

e. Use your answer from part (d) to compute $P(X \geq 2)$.