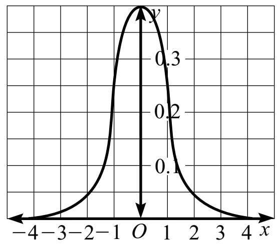
Name ______________________________ Date ______________

CHAPTER 6 The Standard Normal Curve

A normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$ is called the **standard normal curve.** The standard normal curve is used when we are only concerned with how many standard deviations an observation is from the mean.

The graph of the standard normal curve is approximated using the following equation:

$$y = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$



We will use this equation and our graphing utility to explore various areas under the graph of this equation.

EXAMPLE 1 Graph the standard normal curve

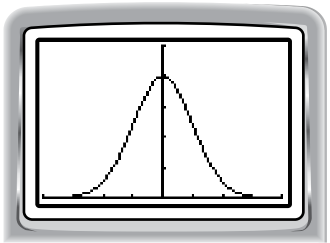
Graph the standard normal curve on a calculator.

Solution:

Using a graphing utility, we enter this equation into y_1. Keeping in mind that the total area under a normal curve will always equal 1, we can choose an appropriate viewing window:

WINDOW:

Xmin = −4
Xmax = 4
Xscl = 1
Ymin = 0
Ymax = 0.5
Yscl = 0.1
Xres = 1



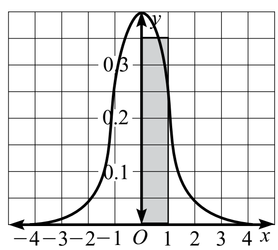
The standard normal curve graphed on the calculator matches the graph of the standard normal curve shown above. ■

We know that in any normal distribution, about 34% of the data lies between the mean μ and one standard deviation to the right of μ. We will to use the equation for a standard normal curve along with rectangles to approximate the area under the standard normal curve between 0 and 1.

EXAMPLE 2 Use one and two rectangles to approximate area

Approximate the area under the standard normal curve between 0 and one standard deviation using one rectangle and two rectangles.

Solution:



The graph shows a rectangle of width 1. The height (h) of the rectangle equals the value of the function at the midpoint of the interval. The midpoint of the interval is 0.5.

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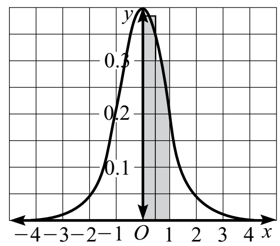
Using the table feature on the graphing utility, we find the value of the standard normal curve at $x = 0.5$ is approximately equal to 0.352. So, the area of the rectangle is approximately equal to $1(0.352) = 0.352$. This is an over-estimate of the expected area of 0.34.

We can obtain a better approximation using a smaller rectangle width. Choosing a rectangle width of 0.5, we obtain two rectangles.



The heights of the rectangles are equal to the values of the function at the midpoint of each interval. Again, we can use the table feature on the graphing utility to find the heights (h_1 and h_2) of the rectangles at $x = 0.25$ and $x = 0.75$. If A_1 and A_2 represent the areas of the rectangles, then the area can be approximated by summing the two areas:

$$A_1 + A_2 = 0.5(0.387) + 0.5(0.301) \approx 0.344$$

This gives us a closer approximation of the true area under the standard normal curve from 0 to 1. ■

Practice

Use midpoints to determine the height of the rectangles.

1. From $x = 0$ to $x = 2$ on the standard normal curve,

a. draw rectangles with width 0.5 that can be used to approximate the area under the standard normal curve over this interval.

b. calculate the area of each rectangle to find the approximate area under the curve.

2. From $x = -1$ to $x = 1$ on the standard normal curve,

a. draw rectangles with width 0.5 that can be used to approximate the area under the standard normal curve over this interval.

b. calculate the area of each rectangle to find the approximate area under the curve.

3. From $x = 0$ to $x = 1$ on the standard normal curve,

a. draw rectangles with width 0.25 that can be used to approximate the area under the standard normal curve over this interval.

b. calculate the area of each rectangle to find the approximate area under the curve.

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