

**CHAPTER
6****Adding Equals to Equals**

The reflexive, symmetric, and transitive properties of equality are not the only properties of equality that are used to find solutions to equations. Here is another property that is used frequently.

KEY CONCEPT**The Addition Property of Equality**

If $a = b$, then $a + c = b + c$.

This is the property that allows us to add the same thing to both sides of an equation.

EXAMPLE 1 **Solve equations using the addition property of equality**

Solve each of the following equations.

a. $x - 7 = 12$

b. $-3 = x + 9$

Solution:

a. $x - 7 = 12$
 $x - 7 + 7 = 12 + 7$
 $x = 19$

Add 7 to both sides.
Simplify.

b. $-3 = x + 9$
 $-3 + (-9) = x + 9 + (-9)$
 $-12 = x$
 $x = -12$

Add -9 to both sides.
Simplify.
Symmetric property of equality ■

Part (b) illustrates how the addition property of equality can be used to justify *subtracting* the same quantity (in this case, 9) from both sides of an equation, because subtracting a number is the same as adding its opposite.

The addition property of equality can be combined with the substitution property of equality to get the following result of adding equals to equals.

If $a = b$ and $c = d$, then $a + c = b + d$.

This can be shown as follows.

$a = b$	Given
$a + c = b + c$	Addition property of equality
$a + c = b + d$	Substitution property of equality

This result is the basis for the elimination method of solving linear systems introduced in Lesson 7.3.

EXAMPLE 2 **Add equals to equals and use the elimination method**

Explain how adding equals to equals makes it possible to solve the following linear system by the elimination method.

CHAPTER
6
Adding Equals to Equals *continued*

$$\begin{aligned} 5x + 3y &= 13 \\ -2x - 3y &= 11 \end{aligned}$$

Solution:

Since $5x + 3y = 13$ and $-2x - 3y = 11$, we know by adding equals to equals that $(5x + 3y) + (-2x - 3y) = 13 + 11$. By adding and simplifying,

$$\begin{array}{r} 5x + 3y = 13 \\ -2x - 3y = 11 \\ \hline 3x \qquad = 24 \end{array}$$

It follows that $x = 8$. Substituting back into either equation and solving for y yields $y = -9$. ■

The other version of the elimination method, where equations are subtracted instead of added, is justified by a combination of adding equals to equals and another property, the multiplication property of equality (see Exercise 4).

Practice

- Tickets to a school fair are \$3 per adult and \$1 per student. A total of 1395 people attend, and the ticket sales amount to \$2867.
 - Write a linear system that could be solved to find the number of adults and the number of students who attend the fair.
 - Suppose the system is solved by subtracting the equation that describes ticket sales from the equation that describes attendance numbers. Does the resulting equation have an interesting interpretation? Why or why not?
 - Solve the system to find the number of adults and the number of students who attended the fair.
- Writing** Suppose someone decided to solve the given linear system by adding the equations instead of subtracting.

$$\begin{aligned} 8x + 7y &= 22 \\ -5x + 7y &= 9 \end{aligned}$$

Would this method be successful? Why or why not?
- Writing** Suppose someone decided to solve the linear system by switching the two sides of the second equation and then adding the result to the first equation.

$$\begin{aligned} -3x + 4y &= -2 \\ 7x + 4y &= -10 \end{aligned}$$

Would this method be successful? Why or why not?
- Challenge** According to the Multiplication Property of Equality, if $a = b$ and $c = d$, then $ac = bd$. Explain how the Multiplication Property can be used, together with the Addition Property, to justify the subtraction version of the Elimination Method for solving linear systems. (Hint: Remember that subtracting a number is the same as adding its opposite.)