Name ______________________________ Date ______________

LESSON 6.3

Study Guide

For use with the lesson "Use Normal Distributions"

GOAL **Study normal distributions.**

Vocabulary

A **normal distribution** is modeled by a bell-shaped curve called a **normal curve** that is symmetric about the mean.

The **standard normal distribution** is the normal distribution with mean 0 and standard deviation 1. The formula below can be used to transform x-values from a normal distribution with mean $\bar{x}$ and standard deviation σ into z-values having a standard normal distribution.

$$z = \frac{x - \bar{x}}{\sigma}$$

The z-value for a particular x-value is called the ***z*-score** for the x-value and is the number of standard deviations the x-value lies above or below the mean $\bar{x}$.

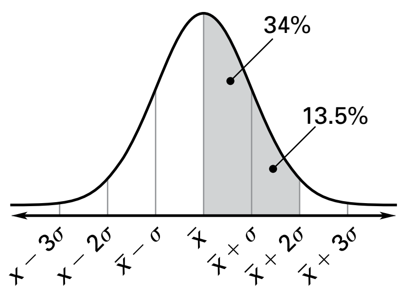
EXAMPLE 1 Find a normal probability

A normal distribution has mean $\bar{x}$ and standard deviation σ. For a randomly selected x-value from the distribution, find $P(\bar{x} \le x \le \bar{x} + 2\sigma)$.

Solution

The probability that a randomly selected x-value lies between $\bar{x}$ and $\bar{x} + 2\sigma$ is the shaded area under the normal curve shown.

$$P(\bar{x} \le x \le \bar{x} + 2\sigma) = 0.34 + 0.135$$
$$= 0.475$$



Exercises for Example 1

A normal distribution has mean $\bar{x}$ and standard deviation σ. Find the indicated probability for a randomly selected x-value from the distribution.

1. $P(x \le \bar{x} + \sigma)$

2. $P(x \ge \bar{x} + \sigma)$

3. $P(\bar{x} \le x \le \bar{x} + \sigma)$

4. $P(\bar{x} \le x \le \bar{x} + 3\sigma)$

5. $P(x \le \bar{x} + 2\sigma)$

6. $P(x \ge \bar{x} - \sigma)$

LESSON 6.3

Study Guide *continued*

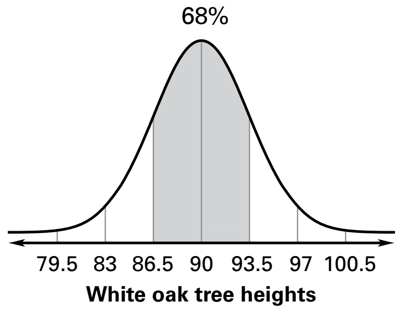
For use with the lesson "Use Normal Distributions"

EXAMPLE 2 Interpret normally distributed data

Oak Trees The heights (in feet) of fully grown white oak trees are normally distributed with a mean of 90 feet and a standard deviation of 3.5 feet. About what percent of white oak trees have heights between 86.5 feet and 93.5 feet?

Solution

The heights of 86.5 feet and 93.5 feet represent one standard deviation on either side of the mean, as shown. So, 68% of the trees have heights between 86.5 feet and 93.5 feet.



EXAMPLE 3 Use a *z*-score and the standard normal table

In Example 2, find the probability that a randomly selected white oak tree has a height of at most 94 feet.

Solution

STEP 1 **Find** the z-score corresponding to an x-value of 94.

$$z = \frac{x - \bar{x}}{\sigma} = \frac{94 - 90}{3.5} \approx 1.1$$

STEP 2 **Use** the standard normal table on page 759 of your textbook to find $P(x \le 94) \le P(z \le 1.1)$.

The table shows that $P(z \le 1.1) = 0.8643$. So, the probability that a randomly selected white oak tree has a height of at most 94 feet is about 0.8643.

z	**.0**	**.1**	**.2**
−3	.0013	.0010	.0007
−2	.0228	.0179	.0139
−1	.1587	.1357	.1151
−0	.5000	.4602	.4207
0	.5000	.5398	.5793
1	.8413	.8643	.8849

Exercises for Examples 2 and 3

In the following exercises, refer to Example 2.

7. About what percent of white oak trees have heights below 97 feet?

8. About what percent of white oak trees have heights between 83 feet and 90 feet?

9. Find the probability that a randomly selected white oak tree has a height of at most 85 feet.