6.3 Solve Linear Systems by Adding or Subtracting

Before You solved linear systems by graphing and using substitution.

Now You will solve linear systems using elimination.

Why? So you can solve a problem about arranging flowers, as in Ex. 42.

Key Vocabulary
- **system of linear equations**

When solving a linear system, you can sometimes add or subtract the equations to obtain a new equation in one variable. This method is called *elimination*.

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

KEY CONCEPT *For Your Notebook*

Solving a Linear System Using the Elimination Method

STEP 1 **Add or subtract** the equations to eliminate one variable.

STEP 2 **Solve** the resulting equation for the other variable.

STEP 3 **Substitute** in either original equation to find the value of the eliminated variable.

EXAMPLE 1 Use addition to eliminate a variable

Solve the linear system: $2x + 3y = 11$ Equation 1; $-2x + 5y = 13$ Equation 2

Solution

ADD EQUATIONS
When the coefficients of one variable are opposites, add the equations to eliminate the variable.

STEP 1 **Add** the equations to eliminate one variable.

$$\begin{array}{r} 2x + 3y = 11 \\ -2x + 5y = 13 \\ \hline 8y = 24 \end{array}$$

STEP 2 **Solve** for y.

$$y = 3$$

STEP 3 **Substitute** 3 for y in either equation and solve for x.

$2x + 3y = 11$ **Write Equation 1.**

$2x + 3(3) = 11$ **Substitute 3 for y.**

$x = 1$ **Solve for x.**

▶ The solution is $(1, 3)$.

CHECK Substitute 1 for x and 3 for y in each of the original equations.

$2x + 3y = 11$

$2(1) + 3(3) \stackrel{?}{=} 11$

$11 = 11$ ✓

$-2x + 5y = 13$

$-2(1) + 5(3) \stackrel{?}{=} 13$

$13 = 13$ ✓

EXAMPLE 2 Use subtraction to eliminate a variable

Solve the linear system: $4x + 3y = 2$ Equation 1
$5x + 3y = -2$ Equation 2

Solution

SUBTRACT EQUATIONS
When the coefficients of one variable are the same, subtract the equations to eliminate the variable.

STEP 1 **Subtract** the equations to eliminate one variable.

$$\begin{array}{r} 4x + 3y = 2 \\ 5x + 3y = -2 \\ \hline -x \quad\quad = 4 \end{array}$$

STEP 2 **Solve** for x.

$$x = -4$$

STEP 3 **Substitute** -4 for x in either equation and solve for y.

$4x + 3y = 2$ Write Equation 1.

$4(-4) + 3y = 2$ Substitute -4 for x.

$y = 6$ Solve for y.

▶ The solution is $(-4, 6)$.

EXAMPLE 3 Arrange like terms

Solve the linear system: $8x - 4y = -4$ Equation 1
$4y = 3x + 14$ Equation 2

Solution

AVOID ERRORS
Make sure that the equal signs are in the same column, just as the like terms are.

STEP 1 **Rewrite** Equation 2 so that the like terms are arranged in columns.

$$\begin{array}{l} 8x - 4y = -4 \\ 4y = 3x + 14 \end{array} \quad \rightarrow \quad \begin{array}{r} 8x - 4y = -4 \\ -3x + 4y = 14 \end{array}$$

STEP 2 **Add** the equations.

$$\begin{array}{r} 5x \quad\quad = 10 \end{array}$$

STEP 3 **Solve** for x.

$$x = 2$$

STEP 4 **Substitute** 2 for x in either equation and solve for y.

$4y = 3x + 14$ Write Equation 2.

$4y = 3(2) + 14$ Substitute 2 for x.

$y = 5$ Solve for y.

▶ The solution is $(2, 5)$.

GUIDED PRACTICE for Examples 1, 2, and 3

Solve the linear system.

1. $4x - 3y = 5$
$-2x + 3y = -7$

2. $-5x - 6y = 8$
$5x + 2y = 4$

3. $6x - 4y = 14$
$-3x + 4y = 1$

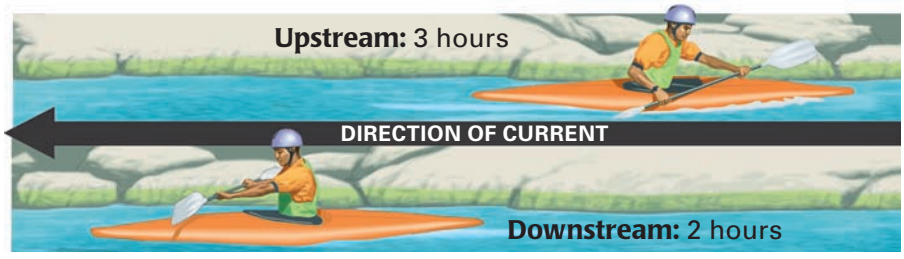
4. $7x - 2y = 5$
$7x - 3y = 4$

5. $3x + 4y = -6$
$2y = 3x + 6$

6. $2x + 5y = 12$
$5y = 4x + 6$

EXAMPLE 4 Write and solve a linear system

KAYAKING During a kayaking trip, a kayaker travels 12 miles upstream (against the current) and 12 miles downstream (with the current), as shown. The speed of the current remained constant during the trip. Find the average speed of the kayak in still water and the speed of the current.



STEP 1 **Write** a system of equations. First find the speed of the kayak going upstream and the speed of the kayak going downstream.

Upstream: $d = rt$ **Downstream:** $d = rt$

$12 = r \cdot 3$ $12 = r \cdot 2$

$4 = r$ $6 = r$

Use the speeds to write a linear system. Let x be the average speed of the kayak in still water, and let y be the speed of the current.

COMBINE SPEEDS
When you go upstream, the speed at which you can travel in still water is decreased by the speed of the current. The opposite is true when you go downstream.

Equation 1: Going upstream

Speed of kayak in still water		Speed of current		Speed of kayak going upstream
x	$-$	y	$=$	4

Equation 2: Going downstream

Speed of kayak in still water		Speed of current		Speed of kayak going downstream
x	$+$	y	$=$	6

STEP 2 **Solve** the system of equations.

$x - y = 4$ **Write Equation 1.**

$x + y = 6$ **Write Equation 2.**

$2x = 10$ **Add equations.**

$x = 5$ **Solve for x.**

Substitute 5 for x in Equation 2 and solve for y.

$5 + y = 6$ **Substitute 5 for x in Equation 2.**

$y = 1$ **Subtract 5 from each side.**

▶ The average speed of the kayak in still water is 5 miles per hour, and the speed of the current is 1 mile per hour.

Animated Algebra at my.hrw.com

GUIDED PRACTICE for Example 4

7. WHAT IF? In Example 4, suppose it takes the kayaker 5 hours to travel 10 miles upstream and 2 hours to travel 10 miles downstream. The speed of the current remains constant during the trip. Find the average speed of the kayak in still water and the speed of the current.

6.3 EXERCISES

HOMEWORK KEY

- ○ = See **WORKED-OUT SOLUTIONS** Exs. 17 and 41
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 22, 36, and 44
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 42

SKILL PRACTICE

1. VOCABULARY Give an example of a linear system in two variables that can be solved by first adding the equations to eliminate one variable.

2. ★ WRITING *Explain* how to solve the linear system shown using the elimination method.

$2x - y = 2$ Equation 1
$2x + 3y = 22$ Equation 2

EXAMPLE 1 for Exs. 3–8

USING ADDITION **Solve the linear system using elimination.**

3. $x + 2y = 13$, $-x + y = 5$

4. $9x + y = 2$, $-4x - y = -17$

5. $-3x - y = 8$, $7x + y = -12$

6. $3x - y = 30$, $-3x + 7y = 6$

7. $-9x + 4y = -17$, $9x - 6y = 3$

8. $-3x - 5y = -7$, $-4x + 5y = 14$

EXAMPLE 2 for Exs. 9–15

USING SUBTRACTION **Solve the linear system using elimination.**

9. $x + y = 1$, $-2x + y = 4$

10. $x - y = -4$, $x + 3y = 4$

11. $2x - y = 7$, $2x + 7y = 31$

12. $6x + y = -10$, $5x + y = -10$

13. $5x + 6y = 50$, $-x + 6y = 26$

14. $4x - 9y = -21$, $4x + 3y = -9$

15. ★ MULTIPLE CHOICE Which ordered pair is a solution of the linear system $4x + 9y = -2$ and $11x + 9y = 26$?

Ⓐ $(-2, 4)$ Ⓑ $(2, -4)$ Ⓒ $(4, -2)$ Ⓓ $(4, 2)$

EXAMPLE 3 for Exs. 16–22

ARRANGING LIKE TERMS **Solve the linear system using elimination.**

16. $2x - y = 32$, $y - 5x = 13$

17. $-8y + 6x = 36$, $6x - y = 15$

18. $2x - y = -11$, $y = -2x - 13$

19. $-x - y = 14$, $x = 5y - 38$

20. $11y - 3x = 18$, $-3x = -16y + 33$

21. $-5x + y = -23$, $-y = 3x - 9$

22. ★ MULTIPLE CHOICE Which ordered pair is a solution of the linear system $2x + y = 10$ and $3y = 2x + 6$?

Ⓐ $(-3, -4)$ Ⓑ $(3, 4)$ Ⓒ $(-4, 3)$ Ⓓ $(4, 3)$

ERROR ANALYSIS *Describe* **and correct the error in finding the value of one of the variables in the given linear system.**

23. $5x - 7y = 16$
$-x - 7y = 8$

$5x - 7y = 16$
$-x - 7y = 8$
$4x = 24$
$x = 6$

24. $3x - 2y = -3$
$5y = 60 - 3x$

$3x - 2y = -3$
$-3x + 5y = 60$
$3y = 57$
$y = 19$

SOLVING LINEAR SYSTEMS **Solve the linear system using elimination.**

25. $-x + \frac{1}{2}y = -19$
$x - y = 12$

26. $\frac{1}{4}x - \frac{2}{3}y = 7$
$\frac{1}{2}x + \frac{2}{3}y = -4$

27. $8x - \frac{1}{2}y = -38$
$\frac{1}{4}x - \frac{1}{2}y = -7$

28. $5.2x + 3.5y = 54$
$-3.6x + 3.5y = 10$

29. $1.3x - 3y = -17.6$
$-1.3x + 4.5y = 25.1$

30. $-2.6x - 3.2y = 4.8$
$1.9x - 3.2y = -4.2$

31. $\frac{4}{5}x + \frac{2}{5}y = 14$
$\frac{2}{5}y + \frac{1}{5}x = 11$

32. $2.7x + 1.5y = 36$
$3.5y = 2.7x - 6$

33. $4 - 4.8x = 1.7y$
$12.8 + 1.7y = -13.2x$

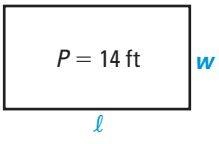
34. WRITING AN EQUATION OF A LINE Use the following steps to write an equation of the line that passes through the points (1, 2) and (−4, 12).

a. Write a system of linear equations by substituting 1 for x and 2 for y in $y = mx + b$ and −4 for x and 12 for y in $y = mx + b$.

b. Solve the system of linear equations from part (a). What is the slope of the line? What is the y-intercept?

c. Write an equation of the line that passes through (1, 2) and (−4, 12).

35. GEOMETRY The rectangle has a perimeter P of 14 feet, and twice its length ℓ is equal to 1 less than 4 times its width w. Write and solve a system of linear equations to find the length and the width of the rectangle.



36. ★ SHORT RESPONSE Find the solution of the system of linear equations below. *Explain* your steps.

$x + 3y = 8$ **Equation 1**
$x - 6y = -19$ **Equation 2**
$5x - 3y = -14$ **Equation 3**

37. CHALLENGE For $a \neq 0$, what is the solution of the system $ax + 2y = 4$ and $ax - 3y = -6$?

38. CHALLENGE Solve for x, y, and z in the system of equations below. *Explain* your steps.

$x + 7y + 3z = 29$ **Equation 1**
$3z + x - 2y = -7$ **Equation 2**
$5y = 10 - 2x$ **Equation 3**

○ = See WORKED-OUT SOLUTIONS in Student Resources ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

PROBLEM SOLVING

EXAMPLE 4 for Exs. 39–41

39. ROWING During a practice, a 4 person crew team rows a rowing shell upstream (against the current) and then rows the same distance downstream (with the current). The shell moves upstream at a speed of 4.3 meters per second and downstream at a speed of 4.9 meters per second. The speed of the current remains constant. Use the models below to write and solve a system of equations to find the average speed of the shell in still water and the speed of the current.

Upstream

Speed of shell in still water	−	Speed of current	=	Speed of shell

Downstream

Speed of shell in still water	+	Speed of current	=	Speed of shell

40. OIL CHANGE Two cars get an oil change at the same service center. Each customer is charged a fee x (in dollars) for the oil change plus y dollars per quart of oil used. The oil change for the car that requires 5 quarts of oil costs $22.45. The oil change for the car that requires 7 quarts of oil costs $25.45. Find the fee and the cost per quart of oil.

41. PHONES Cellular phone ring tones can be monophonic or polyphonic. Monophonic ring tones play one tone at a time, and polyphonic ring tones play multiple tones at a time. The table shows the ring tones downloaded from a website by two customers. Use the information to find the cost of a monophonic ring tone and a polyphonic ring tone, assuming that all monophonic ring tones cost the same and all polyphonic ring tones cost the same.

Customer	Monophonic ring tones	Polyphonic ring tones	Total cost (dollars)
Julie	3	2	12.85
Tate	1	2	8.95

42. MULTIPLE REPRESENTATIONS For a floral arrangement class, Alicia has to create an arrangement of twigs and flowers that has a total of 9 objects. She has to pay for the twigs and flowers that she uses in her arrangement. Each twig costs $1, and each flower costs $3.

a. Writing a System Alicia spends $15 on the twigs and flowers. Write and solve a linear system to find the number of twigs and the number of flowers she used.

b. Making a Table Make a table showing the number of twigs in the arrangement and the total cost of the arrangement when the number of flowers purchased is 0, 1, 2, 3, 4, or 5. Use the table to check your answer to part (a).

43. MULTI-STEP PROBLEM On a typical day with light winds, the 1800 mile flight from Charlotte, North Carolina, to Phoenix, Arizona, takes longer than the return trip because the plane has to fly into the wind.

a. The flight from Charlotte to Phoenix is 4 hours 30 minutes long, and the flight from Phoenix to Charlotte is 4 hours long. Find the average speed (in miles per hour) of the airplane on the way to Phoenix and on the return trip to Charlotte.

b. Let s be the speed (in miles per hour) of the plane with no wind, and let w be the speed (in miles per hour) of the wind. Use your answer to part (a) to write and solve a system of equations to find the speed of the plane with no wind and the speed of the wind.

44. ★ SHORT RESPONSE The students in the graduating classes at the three high schools in a school district have to pay for their caps and gowns. A cap-and-gown set costs x dollars, and an extra tassel costs y dollars. At one high school, students pay \$3262 for 215 cap-and-gown sets and 72 extra tassels. At another high school, students pay \$3346 for 221 cap-and-gown sets and 72 extra tassels. How much will students at the third high school pay for 218 cap-and-gown sets and 56 extra tassels? *Explain.*

45. CHALLENGE A clothing manufacturer makes men's dress shirts. For the production process, an ideal sleeve length x (in centimeters) for each shirt size and an allowable deviation y (in centimeters) from the ideal length are established. The deviation is expressed as $\pm y$. For a specific shirt size, the minimum allowable sleeve length is 62.2 centimeters and the maximum allowable sleeve length is 64.8 centimeters. Find the ideal sleeve length and the allowable deviation.