6.4 Solve Linear Systems by Multiplying First

Before You solved linear systems by adding or subtracting.

Now You will solve linear systems by multiplying first.

Why So you can solve a problem about preparing food, as in Ex. 39.

Key Vocabulary
- **least common multiple**

In a linear system like the one below, neither variable can be eliminated by adding or subtracting the equations. For systems like these, you can multiply one or both of the equations by a constant so that adding or subtracting the equations will eliminate one variable.

$5x + 2y = 16 \xrightarrow{\times 2} 10x + 4y = 32$

$3x - 4y = 20 \longrightarrow 3x - 4y = 20$

The new system is equivalent to the original system.

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

EXAMPLE 1 Multiply one equation, then add

Solve the linear system:

$6x + 5y = 19$ Equation 1

$2x + 3y = 5$ Equation 2

Solution

STEP 1 **Multiply** Equation 2 by -3 so that the coefficients of x are opposites.

$6x + 5y = 19$

$2x + 3y = 5 \xrightarrow{\times (-3)} -6x - 9y = -15$

STEP 2 **Add** the equations. $-4y = 4$

STEP 3 **Solve** for y. $y = -1$

STEP 4 **Substitute** -1 for y in either of the original equations and solve for x.

$2x + 3y = 5$ Write Equation 2.

$2x + 3(-1) = 5$ Substitute -1 for y.

$2x + (-3) = 5$ Multiply.

$2x = 8$ Subtract -3 from each side.

$x = 4$ Divide each side by 2.

▶ The solution is $(4, -1)$.

CHECK Substitute 4 for x and -1 for y in each of the original equations.

Equation 1	Equation 2
$6x + 5y = 19$	$2x + 3y = 5$
$6(4) + 5(-1) \stackrel{?}{=} 19$	$2(4) + 3(-1) \stackrel{?}{=} 5$
$19 = 19$ ✓	$5 = 5$ ✓

ANOTHER WAY
You can also multiply Equation 2 by 3 and subtract the equations.

MULTIPLYING BOTH EQUATIONS To eliminate one variable when adding or subtracting equations in a linear system, you may need to multiply both equations by constants. Use the least common multiple of the coefficients of one of the variables to determine the constants.

$2x - 9y = 1$ $\xrightarrow{\times 4}$ $8x - 36y = 4$

$7x - 12y = 23$ $\xrightarrow{\times 3}$ $21x - 36y = 69$

The least common multiple of -9 and -12 is -36.

EXAMPLE 2 Multiply both equations, then subtract

Solve the linear system:

$4x + 5y = 35$ Equation 1

$2y = 3x - 9$ Equation 2

Solution

STEP 1 **Arrange** the equations so that like terms are in columns.

$4x + 5y = 35$ Write Equation 1.

$-3x + 2y = -9$ Rewrite Equation 2.

ANOTHER WAY
You can also multiply Equation 1 by 3 and Equation 2 by 4. Then add the revised equations to eliminate x.

STEP 2 **Multiply** Equation 1 by 2 and Equation 2 by 5 so that the coeffcient of y in each equation is the least common multiple of 5 and 2, or 10.

$4x + 5y = 35$ $\xrightarrow{\times 2}$ $8x + 10y = 70$

$-3x + 2y = -9$ $\xrightarrow{\times 5}$ $-15x + 10y = -45$

STEP 3 **Subtract** the equations. $23x = 115$

STEP 4 **Solve** for x. $x = 5$

STEP 5 **Substitute** 5 for x in either of the original equations and solve for y.

$4x + 5y = 35$ Write Equation 1.

$4(5) + 5y = 35$ Substitute 5 for x.

$y = 3$ Solve for y.

▶ The solution is $(5, 3)$.

CHECK Substitute 5 for x and 3 for y in each of the original equations.

Equation 1	Equation 2
$4x + 5y = 35$	$2y = 3x - 9$
$4(5) + 5(3) \stackrel{?}{=} 35$	$2(3) \stackrel{?}{=} 3(5) - 9$
$35 = 35$ ✓	$6 = 6$ ✓

Animated Algebra at my.hrw.com

GUIDED PRACTICE for Examples 1 and 2

Solve the linear system using elimination.

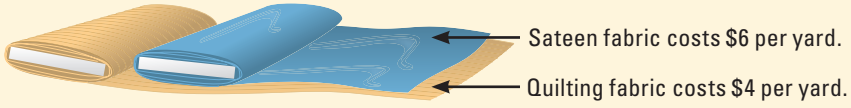
1. $6x - 2y = 1$
$-2x + 3y = -5$

2. $2x + 5y = 3$
$3x + 10y = -3$

3. $3x - 7y = 5$
$9y = 5x + 5$

EXAMPLE 3 Standardized Test Practice

Darlene is making a quilt that has alternating stripes of regular quilting fabric and sateen fabric. She spends \$76 on a total of 16 yards of the two fabrics at a fabric store. Which system of equations can be used to find the amount x (in yards) of regular quilting fabric and the amount y (in yards) of sateen fabric she purchased?



ELIMINATE CHOICES
You can eliminate choice A because $x + y$ cannot equal both 16 and 76.

Ⓐ $x + y = 16$
$x + y = 76$

Ⓑ $x + y = 16$
$4x + 6y = 76$

Ⓒ $x + y = 76$
$4x + 6y = 16$

Ⓓ $x + y = 16$
$6x + 4y = 76$

Solution

Write a system of equations where x is the number of yards of regular quilting fabric purchased and y is the number of yards of sateen fabric purchased.

Equation 1: Amount of fabric

Amount of quilting fabric	+	Amount of sateen fabric	=	Total yards of fabric
↓		↓		↓
x	+	y	=	16

Equation 2: Cost of fabric

Quilting fabric price (dollars/yd)	·	Amount of quilting fabric (yd)	+	Sateen fabric price (dollars/yd)	·	Amount of sateen fabric (yd)	=	Total cost (dollars)
↓		↓		↓		↓		↓
4	·	x	+	6	·	y	=	76

The system of equations is:
$x + y = 16$ **Equation 1**
$4x + 6y = 76$ **Equation 2**

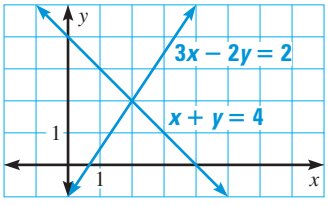
▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Example 3

4. **SOCCER** A sports equipment store is having a sale on soccer balls. A soccer coach purchases 10 soccer balls and 2 soccer ball bags for \$155. Another soccer coach purchases 12 soccer balls and 3 soccer ball bags for \$189. Find the cost of a soccer ball and the cost of a soccer ball bag.

CONCEPT SUMMARY *For Your Notebook*

Methods for Solving Linear Systems

Method	Example	When to Use
Table	x | $y = 2x$ | $y = 3x - 1$ 0 | 0 | −1 1 | 2 | 2 2 | 4 | 5	When x-values are integers, so that equal values can be seen in the table
Graphing		When you want to see the lines that the equations represent
Substitution	$\mathbf{y = 4 - 2x}$ $4x + 2y = 8$	When one equation is already solved for x or y
Addition	$4x + \mathbf{7y} = 15$ $6x - \mathbf{7y} = 5$	When the coefficients of one variable are opposites
Subtraction	$\mathbf{3x} + 5y = -13$ $\mathbf{3x} + y = -5$	When the coefficients of one variable are the same
Multiplication	$9x + 2y = 38$ $3x - 5y = 7$	When no corresponding coefficients are the same or opposites

6.4 EXERCISES

HOMEWORK KEY

- ○ = See **WORKED-OUT SOLUTIONS** Exs. 15 and 39
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 34, 41, and 42
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 40

SKILL PRACTICE

1. **VOCABULARY** What is the least common multiple of 12 and 18?

2. ★ **WRITING** *Explain* how to solve the linear system using the elimination method.

$$2x - 3y = -4 \quad \text{Equation 1}$$
$$7x + 9y = -5 \quad \text{Equation 2}$$

EXAMPLE 1 for Exs. 3–8

SOLVING LINEAR SYSTEMS **Solve the linear system using elimination.**

3. $x + y = 2$
   $2x + 7y = 9$

4. $3x - 2y = 3$
   $-x + y = 1$

5. $4x + 3y = 8$
   $x - 2y = 13$

6. $10x - 9y = 46$
   $-2x + 3y = 10$

7. $8x - 5y = 11$
   $4x - 3y = 5$

8. $11x - 20y = 28$
   $3x + 4y = 36$

EXAMPLE 2
Exs. 9–20

SOLVING LINEAR SYSTEMS **Solve the linear system using elimination.**

9. $4x - 3y = 8$
$5x - 2y = -11$

10. $-2x - 5y = 9$
$3x + 11y = 4$

11. $7x - 6y = -1$
$5x - 4y = 1$

12. $7x + 3y = -12$
$2x + 5y = 38$

13. $9x - 8y = 4$
$2x - 3y = -4$

14. $12x - 7y = -2$
$-8x + 11y = 14$

15. $9x + 2y = 39$
$6x + 13y = -9$

16. $-7x + 10y = 11$
$-8x + 15y = 34$

17. $-14x + 15y = 15$
$21x - 20y = -10$

18. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the linear system $15x + 8y = 6$ and $25x + 12y = 14$?

Ⓐ $(-3, -2)$ Ⓑ $(-3, 2)$ Ⓒ $(-2, -3)$ Ⓓ $(2, -3)$

ERROR ANALYSIS ***Describe*** **and correct the error when solving the linear system.**

19.

$2x - 3y = -9 \xrightarrow{\times 2} 4x - 6y = -18$
$5x - 6y = -9 \qquad 5x - 6y = -9$
$9x = -27$
$x = -3$

20.

$9x + 8y = 11 \xrightarrow{\times 3} 27x + 24y = 11$
$7x + 6y = 9 \xrightarrow{\times 4} 28x + 24y = 9$
$-x = 2$
$x = -2$

SOLVING LINEAR SYSTEMS **Solve the linear system using any algebraic method.**

21. $3x + 2y = 4$
$2y = 8 - 5x$

22. $4x - 5y = 18$
$3x = y + 11$

23. $8x - 9y = -15$
$-4x = 19 + y$

24. $0.3x + 0.1y = -0.1$
$-x + y = 3$

25. $4.4x - 3.6y = 7.6$
$x - y = 1$

26. $3x - 2y = -20$
$x + 1.2y = 6.4$

27. $0.2x - 1.5y = -1$
$x - 4.5y = 1$

28. $1.5x - 3.5y = -5$
$-1.2x + 2.5y = 1$

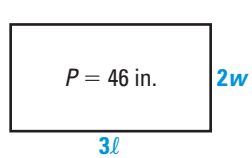
29. $4.9x + 2.4y = 7.4$
$0.7x + 3.6y = -2.2$

30. $x + y = 0$
$\frac{1}{2}x - \frac{1}{2}y = 2$

31. $3x + y = \frac{1}{3}$
$2x - 3y = \frac{8}{3}$

32. $\frac{3}{5}x - \frac{3}{4}y = -3$
$\frac{2}{5}x + \frac{1}{3}y = 8$

33. **GEOMETRY** A rectangle has a perimeter of 18 inches. A new rectangle is formed by doubling the width w and tripling the length ℓ, as shown. The new rectangle has a perimeter P of 46 inches.



a. Write and solve a system of linear equations to find the length and width of the original rectangle.

b. Find the length and width of the new rectangle.

34. ★ **WRITING** For which values of a can you solve the linear system $ax + 3y = 2$ and $4x + 5y = 6$ without multiplying first? *Explain*.

CHALLENGE **Find the values of a and b so that the linear system has the given solution.**

$ax - by = 4$ **Equation 1**
$bx - ay = 10$ **Equation 2**

35. $(4, 2)$

36. $(2, 1)$

PROBLEM SOLVING

EXAMPLE 3 for Exs. 37–39

37. BOOK SALE A library is having a book sale to raise money. Hardcover books cost $4 each and paperback books cost $2 each. A person spends $26 for 8 books. How many hardcover books did she purchase?

38. MUSIC A website allows users to download individual songs or an entire album. All individual songs cost the same to download, and all albums cost the same to download. Ryan pays $14.94 to download 5 individual songs and 1 album. Seth pays $22.95 to download 3 individual songs and 2 albums. How much does the website charge to download a song? an entire album?

39. FARM PRODUCTS The table shows the number of apples needed to make the apple pies and applesauce sold at a farm store. During a recent apple picking at the farm, 169 Granny Smith apples and 95 Golden Delicious apples were picked. How many apple pies and batches of applesauce can be made if every apple is used?

Type of apple	Granny Smith	Golden Delicious
Needed for a pie	5	3
Needed for a batch of applesauce	4	2

40. MULTIPLE REPRESENTATIONS Tickets for admission to a high school football game cost $3 for students and $5 for adults. During one game, $2995 was collected from the sale of 729 tickets.

a. Writing a System Write and solve a system of linear equations to find the number of tickets sold to students and the number of tickets sold to adults.

b. Drawing a Graph Graph the system of linear equations. Use the graph to determine whether your answer to part (a) is reasonable.

41. ★ SHORT RESPONSE A dim sum restaurant offers two sizes of dishes: small and large. All small dishes cost the same and all large dishes cost the same. The bills show the cost of the food before the tip is included. What will 3 small and 2 large dishes cost before the tip is included? *Explain*.

42. ★ OPEN-ENDED *Describe* a real-world problem that can be solved using a system of linear equations. Then solve the problem and explain what the solution means in this situation.

○ = See WORKED-OUT SOLUTIONS in Student Resources ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

43. INVESTMENTS Matt invested $2000 in stocks and bonds. This year the bonds paid 8% interest, and the stocks paid 6% in dividends. Matt received a total of $144 in interest and dividends. How much money did he invest in stocks? in bonds?

44. CHALLENGE You drive a car 45 miles at an average speed r (in miles per hour) to reach your destination. Due to traffic, your average speed on the return trip is $\frac{3}{4}r$. The round trip took a total of 1 hour 45 minutes. Find the average speed for each leg of your trip.

QUIZ

Solve the linear system using elimination.

1. $x + y = 4$
$-3x + y = -8$

2. $2x - y = 2$
$6x - y = -2$

3. $x + y = 5$
$-x + y = -3$

4. $x + 3y = -10$
$-x + 5y = -30$

5. $x + 3y = 10$
$3x - y = 13$

6. $x + 7y = 10$
$x + 2y = -8$

7. $4x - y = -2$
$3x + 2y = 7$

8. $x + 3y = 1$
$5x + 6y = 14$

9. $3x + y = 21$
$x + y = 1$

10. $2x - 3y = -5$
$5x + 2y = 16$

11. $7x + 2y = 13$
$4x + 3y = 13$

12. $\frac{1}{3}x + 5y = -3$
$-\frac{2}{3}x + 6y = -10$

Multiply and Then Add Equations

MATHEMATICAL PRACTICES

Use appropriate tools strategically.

QUESTION How can you see why elimination works as a method for solving linear systems?

You have used elimination to solve systems of linear equations, but you may think that it isn't obvious why this method works. You can do an algebraic proof by replacing the numbers in the system with variables, but this is complicated to do. In this activity, you will graph each equation that you get as you use elimination.

EXAMPLE 1 Solve the linear system using addition

Solve the linear system

$$-2x + y = 1 \quad \text{Equation 1}$$
$$2x + y = 5 \quad \text{Equation 2}$$

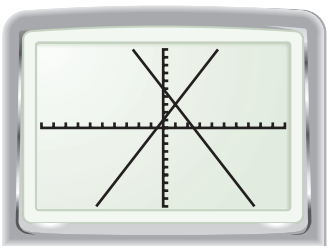
Solution

STEP 1 *Graph the System*

Solve both equations for y.

$$-2x + y = 1 \qquad 2x + y = 5$$
$$y = 1 + 2x \qquad y = 5 - 2x$$

Graph the two equations using a graphing calculator. Notice that the point of intersection of the graphs is the solution of the system.

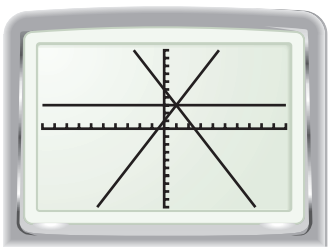


The solution is (1, 3).

STEP 2 *Graph the sum of the equations*

Add the two equations as you would if you were solving the system algebraically. Graph the resulting equation.

$$\begin{array}{rl} -2x + y = 1 & \text{Equation 1} \\ 2x + y = 5 & \text{Equation 2} \\ \hline 2y = 6 & \text{Add.} \\ y = 3 & \text{Solve for } y. \end{array}$$



Now graph the equation $y = 3$ on the same graphing calculator screen with the two original equations.

STEP 3 *Summarize the Results*

All three equations intersect at (1, 3). So, (1, 3) is the solution of the system.

PRACTICE 1

Solve the system using elimination. Graph each resulting equation.

1. $-x + y = 9$
$x + y = 1$

2. $6x - 7y = 4$
$x + 7y = 17$

3. $2x - 3y = 4$
$8x + 3y = 1$

EXAMPLE 2 Solve a linear system using multiplication

Solve the linear system:

$$2x - y = 4 \quad \text{Equation 1}$$
$$-3x + 2y = -7 \quad \text{Equation 2}$$

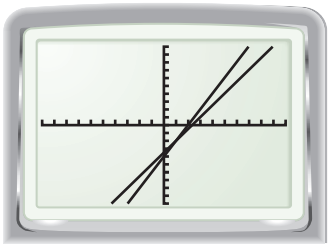
STEP 1 Graph the System

Solve each equation for y.

$2x - y = 4$ $\qquad$ $-3x + 2y = -7$

$y = 2x - 4$ $\qquad$ $y = \dfrac{3x - 7}{2}$

Graph the two equations. The point of intersection of the graphs is the solution of the system.



STEP 2 Use elimination to solve

Multiply each equation by a constant so that you can eliminate a variable x by adding.

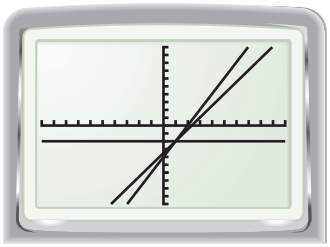
$2x - y = 4$ $\xrightarrow{\times 3}$ $6x - 3y = 12$ **Multiply Equation 1 by 3.**

$-3x + 2y = -7$ $\xrightarrow{\times 2}$ $-6x + 4y = -14$ **Multiply Equation 2 by 2.**

$y = -2$ **Add.**

STEP 3 Graph the resulting equations

Graph the equations $6x - 3y = 12$, $-6x + 4y = -14$, and $y = -2$ on the same graphing calculator screen with the two original equations.



STEP 4 Summarize the Results

All of the equations intersect at $(1, -2)$. So, $(1, -2)$ is the solution of the system.

PRACTICE

Solve the system using elimination. Graph each resulting equation.

4. $x - y = -5$
$4x + 3y = 1$

5. $2x - 5y = 3$
$-x + 2y = -2$

6. $3x + 5y = 3$
$x - y = 9$

7. Solve the linear system using a graphing calculator. Now use a linear combination on the system to eliminate the variable x. Use a linear combination on the system to eliminate the variable y. What do you notice?

$x - 2y = -6$
$2x + y = 8$

DRAW CONCLUSIONS

8. Explain how you could use this method to check whether you have correctly solved a system of linear equations by graphing?

9. Suppose you are trying to solve a system of linear equations that has no solution.

a. What happens when you use the elimination method?

b. What does the graph of the system look like?

c. Will the method of graphing the resulting equations as in Example 2 work with the system?

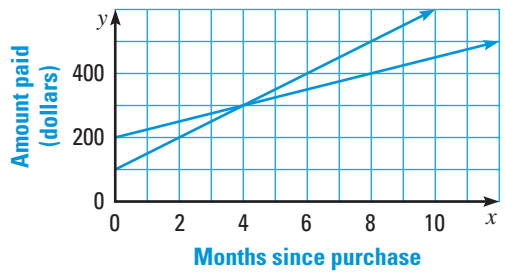
MIXED REVIEW of Problem Solving

MATHEMATICAL PRACTICES Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** Flying into the wind, a helicopter takes 15 minutes to travel 15 kilometers. The return flight takes 12 minutes. The wind speed remains constant during the trip.
   - **a.** Find the helicopter's average speed (in kilometers per hour) for each leg of the trip.
   - **b.** Write a system of linear equations that represents the situation.
   - **c.** What is the helicopter's average speed in still air? What is the speed of the wind?

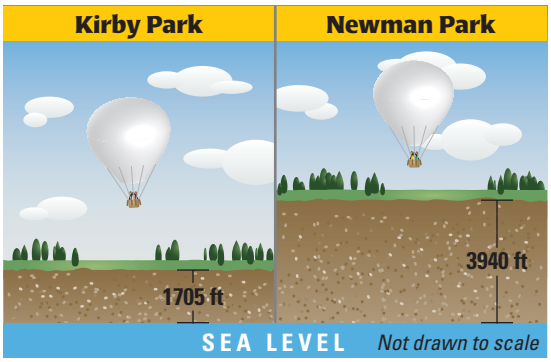
2. **SHORT RESPONSE** At a grocery store, a customer pays a total of $9.70 for 1.8 pounds of potato salad and 1.4 pounds of coleslaw. Another customer pays a total of $6.55 for 1 pound of potato salad and 1.2 pounds of coleslaw. How much do 2 pounds of potato salad and 2 pounds of coleslaw cost? *Explain.*

3. **GRIDDED ANSWER** During one day, two computers are sold at a computer store. The two customers each arrange payment plans with the salesperson. The graph shows the amount y of money (in dollars) paid for the computers after x months. After how many months will each customer have paid the same amount?



4. **OPEN-ENDED** *Describe* a real-world problem that can be modeled by a linear system. Then solve the system and interpret the solution in the context of the problem.

5. **SHORT RESPONSE** A hot air balloon is launched at Kirby Park, and it ascends at a rate of 7200 feet per hour. At the same time, a second hot air balloon is launched at Newman Park, and it ascends at a rate of 4000 feet per hour. Both of the balloons stop ascending after 30 minutes. The diagram shows the altitude of each park. Are the hot air balloons ever at the same height at the same time? *Explain.*



6. **EXTENDED RESPONSE** A chemist needs 500 milliliters of a 20% acid and 80% water mix for a chemistry experiment. The chemist combines x milliliters of a 10% acid and 90% water mix and y milliliters of a 30% acid and 70% water mix to make the 20% acid and 80% water mix.
   - **a.** Write a linear system that represents the situation.
   - **b.** How many milliliters of the 10% acid and 90% water mix and the 30% acid and 70% water mix are combined to make the 20% acid and 80% water mix?
   - **c.** The chemist also needs 500 milliliters of a 15% acid and 85% water mix. Does the chemist need more of the 10% acid and 90% water mix than the 30% acid and 70% water mix to make this new mix? *Explain.*