

Multiply and Then Add Equations



Use appropriate tools strategically.

QUESTION How can you see why elimination works as a method for solving linear systems?

You have used elimination to solve systems of linear equations, but you may think that it isn't obvious why this method works. You can do an algebraic proof by replacing the numbers in the system with variables, but this is complicated to do. In this activity, you will graph each equation that you get as you use elimination.

EXAMPLE 1 Solve the linear system using addition

Solve the linear system

$$\begin{array}{rcl} -2x + y = 1 & \text{Equation 1} \\ 2x + y = 5 & \text{Equation 2} \end{array}$$

Solution

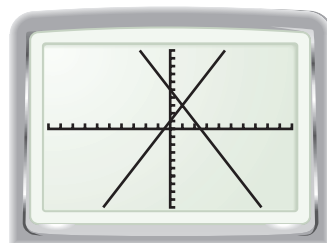
STEP 1 Graph the System

Solve both equations for y .

$$\begin{array}{rcl} -2x + y = 1 & 2x + y = 5 \\ y = 1 + 2x & y = 5 - 2x \end{array}$$

Graph the two equations using a graphing calculator. Notice that the point of intersection of the graphs is the solution of the system.

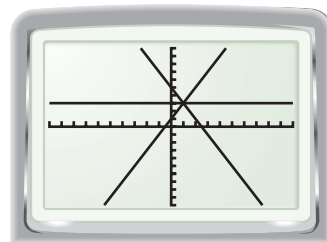
The solution is $(1, 3)$.



STEP 2 Graph the sum of the equations

Add the two equations as you would if you were solving the system algebraically. Graph the resulting equation.

$$\begin{array}{rcl} -2x + y = 1 & \text{Equation 1} \\ 2x + y = 5 & \text{Equation 2} \\ \hline 2y = 6 & \text{Add.} \\ y = 3 & \text{Solve for } y. \end{array}$$



Now graph the equation $y = 3$ on the same graphing calculator screen with the two original equations.

STEP 3 Summarize the Results

All three equations intersect at $(1, 3)$. So, $(1, 3)$ is the solution of the system.

PRACTICE 1

Solve the system using elimination. Graph each resulting equation.

- $-x + y = 9$
 $x + y = 1$
- $6x - 7y = 4$
 $x + 7y = 17$
- $2x - 3y = 4$
 $8x + 3y = 1$

EXAMPLE 2 Solve a linear system using multiplication

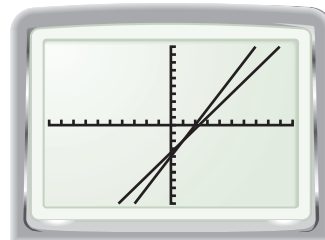
Solve the linear system:

$$\begin{array}{rcl} 2x - y = 4 & \text{Equation 1} \\ -3x + 2y = -7 & \text{Equation 2} \end{array}$$

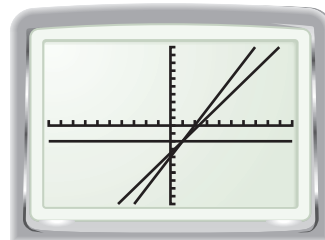
STEP 1 Graph the SystemSolve each equation for y .

$$\begin{array}{rcl} 2x - y = 4 & -3x + 2y = -7 \\ y = 2x - 4 & y = \frac{3x - 7}{2} \end{array}$$

Graph the two equations. The point of intersection of the graphs is the solution of the system.

**STEP 2** Use elimination to solveMultiply each equation by a constant so that you can eliminate a variable x by adding.

$$\begin{array}{rcl} 2x - y = 4 & \times 3 & \rightarrow 6x - 3y = 12 & \text{Multiply Equation 1 by 3.} \\ -3x + 2y = -7 & \times 2 & \rightarrow -6x + 4y = -14 & \text{Multiply Equation 2 by 2.} \\ \hline & & y = -2 & \text{Add.} \end{array}$$

STEP 3 Graph the resulting equationsGraph the equations $6x - 3y = 12$, $-6x + 4y = -14$, and $y = -2$ on the same graphing calculator screen with the two original equations.**STEP 4** Summarize the ResultsAll of the equations intersect at $(1, -2)$. So, $(1, -2)$ is the solution of the system.**PRACTICE**

Solve the system using elimination. Graph each resulting equation.

$$\begin{array}{lll} 4. \ x - y = -5 & 5. \ 2x - 5y = 3 & 6. \ 3x + 5y = 3 \\ \quad 4x + 3y = 1 & \quad -x + 2y = -2 & \quad x - y = 9 \end{array}$$

7. Solve the linear system using a graphing calculator. Now use a linear combination on the system to eliminate the variable x . Use a linear combination on the system to eliminate the variable y . What do you notice?

$$\begin{array}{l} x - 2y = -6 \\ 2x + y = 8 \end{array}$$

DRAW CONCLUSIONS

8. Explain how you could use this method to check whether you have correctly solved a system of linear equations by graphing?
9. Suppose you are trying to solve a system of linear equations that has no solution.
 - a. What happens when you use the elimination method?
 - b. What does the graph of the system look like?
 - c. Will the method of graphing the resulting equations as in Example 2 work with the system?