6.5 Solve Special Types of Linear Systems

Before	You found the solution of a linear system.
Now	You will identify the number of solutions of a linear system.
Why?	So you can compare distances traveled, as in Ex. 39.

Key Vocabulary
- **inconsistent system**
- **consistent dependent system**
- **system of linear equations**
- **parallel**

A linear system can have no solution or infinitely many solutions. A linear system has no solution when the graphs of the equations are parallel. A linear system with no solution is called an **inconsistent system**.

A linear system has infinitely many solutions when the graphs of the equations are the same line. A linear system with infinitely many solutions is called a **consistent dependent system**.

EXAMPLE 1 A linear system with no solution

COMMON CORE

CC.9-12.A.REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Show that the linear system has no solution.

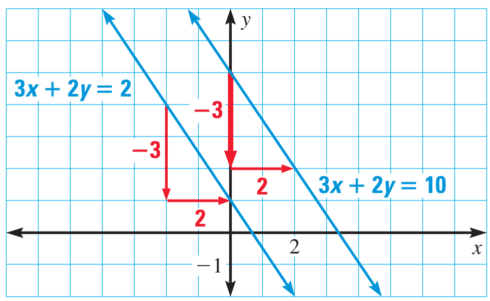
$3x + 2y = 10$ Equation 1
$3x + 2y = 2$ Equation 2

Solution

METHOD 1 Graphing

REVIEW GRAPHING
You may want to review graphing linear equations before graphing linear systems.

Graph the linear system.



▶ The lines are parallel because they have the same slope but different y-intercepts. Parallel lines do not intersect, so the system has no solution.

METHOD 2 Elimination

Subtract the equations.

$$\begin{array}{r} 3x + 2y = 10 \\ 3x + 2y = 2 \\ \hline 0 = 8 \end{array}$$

← This is a false statement.

▶ The variables are eliminated and you are left with a false statement regardless of the values of x and y. This tells you that the system has no solution.

IDENTIFY TYPES OF SYSTEMS
The linear system in Example 1 is called an inconsistent system because the lines do not intersect (are not consistent).

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EXAMPLE 2 A linear system with infinitely many solutions

Show that the linear system has infinitely many solutions.

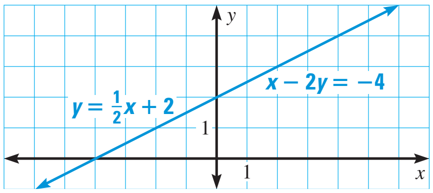
$x - 2y = -4$ **Equation 1**

$y = \frac{1}{2}x + 2$ **Equation 2**

Solution

METHOD 1 Graphing

Graph the linear system.



▶ The equations represent the same line, so any point on the line is a solution. So, the linear system has infinitely many solutions.

METHOD 2 Substitution

Substitute $\frac{1}{2}x + 2$ for y in Equation 1 and solve for x.

$x - 2y = -4$ **Write Equation 1.**

$x - 2\left(\frac{1}{2}x + 2\right) = -4$ **Substitute $\frac{1}{2}x + 2$ for y.**

$-4 = -4$ **Simplify.**

▶ The variables are eliminated and you are left with a statement that is true regardless of the values of x and y. This tells you that the system has infinitely many solutions.

IDENTIFY TYPES OF SYSTEMS
The linear system in Example 2 is called a consistent dependent system because the lines intersect (are consistent) and the equations are equivalent (are dependent).

GUIDED PRACTICE for Examples 1 and 2

Tell whether the linear system has *no solution* or *infinitely many solutions*. *Explain*.

1. $5x + 3y = 6$
$-5x - 3y = 3$

2. $y = 2x - 4$
$-6x + 3y = -12$

IDENTIFYING THE NUMBER OF SOLUTIONS When the equations of a linear system are written in slope-intercept form, you can identify the number of solutions of the system by looking at the slopes and y-intercepts of the lines.

Number of solutions	Slopes and y-intercepts
One solution	Different slopes
No solution	Same slope Different y-intercepts
Infinitely many solutions	Same slope Same y-intercept

EXAMPLE 3 Identify the number of solutions

Without solving the linear system, tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.

a. $5x + y = -2$ Equation 1
$-10x - 2y = 4$ Equation 2

b. $6x + 2y = 3$ Equation 1
$6x + 2y = -5$ Equation 2

Solution

a. $y = -5x - 2$ Write Equation 1 in slope-intercept form.

$y = -5x - 2$ Write Equation 2 in slope-intercept form.

▶ Because the lines have the same slope and the same y-intercept, the system has infinitely many solutions.

b. $y = -3x + \frac{3}{2}$ Write Equation 1 in slope-intercept form.

$y = -3x - \frac{5}{2}$ Write Equation 2 in slope-intercept form.

▶ Because the lines have the same slope but different y-intercepts, the system has no solution.

EXAMPLE 4 Write and solve a system of linear equations

ART An artist wants to sell prints of her paintings. She orders a set of prints for each of two of her paintings. Each set contains regular prints and glossy prints, as shown in the table. Find the cost of one glossy print.

Regular	Glossy	Cost
45	30	$465
15	10	$155

Solution

STEP 1 **Write** a linear system. Let x be the cost (in dollars) of a regular print, and let y be the cost (in dollars) of a glossy print.

$45x + 30y = 465$ Cost of prints for one painting
$15x + 10y = 155$ Cost of prints for other painting

STEP 2 **Solve** the linear system using elimination.

$$\begin{aligned} 45x + 30y &= 465 \\ 15x + 10y &= 155 \end{aligned} \quad \xrightarrow{\times(-3)} \quad \begin{aligned} 45x + 30y &= 465 \\ -45x - 30y &= -465 \\ \hline 0 &= 0 \end{aligned}$$

▶ There are infinitely many solutions, so you cannot determine the cost of one glossy print. You need more information.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Without solving the linear system, tell whether it has *one solution*, *no solution*, or *infinitely many solutions*.

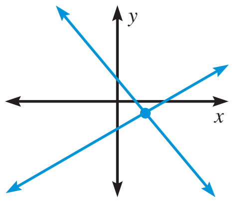
$x - 3y = -15$ Equation 1
$2x - 3y = -18$ Equation 2

4. **WHAT IF?** In Example 4, suppose a glossy print costs $3 more than a regular print. Find the cost of a glossy print.

CONCEPT SUMMARY — *For Your Notebook*

Number of Solutions of a Linear System

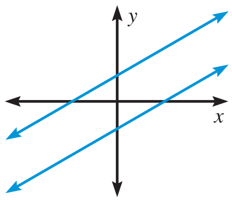
One solution



The lines intersect.

The lines have different slopes.

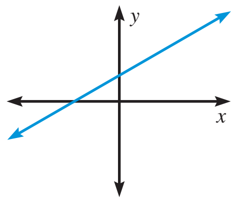
No solution



The lines are parallel.

The lines have the same slope and different *y*-intercepts.

Infinitely many solutions



The lines coincide.

The lines have the same slope and the same *y*-intercept.

6.5 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 11 and 37

★ = **STANDARDIZED TEST PRACTICE** Exs. 3, 4, 24, 25, 32, 33, and 40

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A linear system with no solution is called a(n) __?__ system.

2. **VOCABULARY** Copy and complete: A linear system with infinitely many solutions is called a(n) __?__ system.

3. ★ **WRITING** *Describe* the graph of a linear system that has no solution.

4. ★ **WRITING** *Describe* the graph of a linear system that has infinitely many solutions.

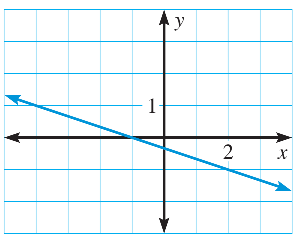
INTERPRETING GRAPHS **Match the linear system with its graph. Then use the graph to tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.**

5. $x - 3y = -9$
   $x - y = -1$

6. $x - y = -4$
   $-3x + 3y = 2$

7. $x + 3y = -1$
   $-2x - 6y = 2$

A.



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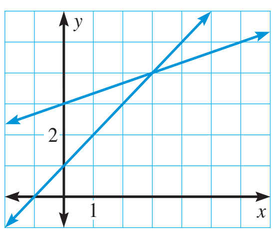

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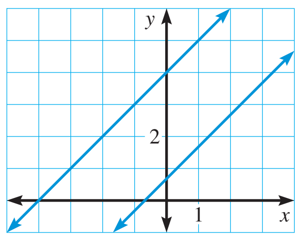


EXAMPLES 1 and 2
for Exs. 8–25

INTERPRETING GRAPHS **Graph the linear system. Then use the graph to tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.**

8. $x + y = -2$
$y = -x + 5$

9. $3x - 4y = 12$
$y = \frac{3}{4}x - 3$

10. $3x - y = -9$
$3x + 5y = -15$

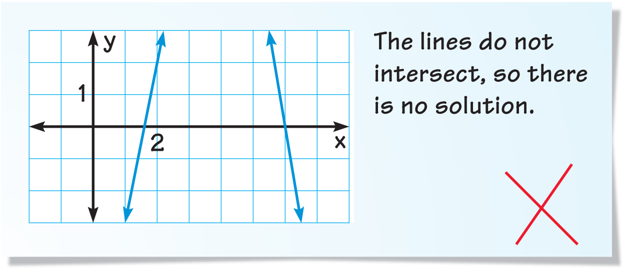
11. $-2x + 2y = -16$
$3x - 6y = 30$

12. $-9x + 6y = 18$
$6x - 4y = -12$

13. $-3x + 4y = 12$
$-3x + 4y = 24$

14. **ERROR ANALYSIS** *Describe* and correct the error in solving the linear system below.

$6x + y = 36$
$5x - y = 8$



SOLVING LINEAR SYSTEMS **Solve the linear system using substitution or elimination.**

15. $2x + 5y = 14$
$6x + 7y = 10$

16. $-16x + 2y = -2$
$y = 8x - 1$

17. $3x - 2y = -5$
$4x + 5y = 47$

18. $5x - 5y = -3$
$y = x + 0.6$

19. $x - y = 0$
$5x - 2y = 6$

20. $x - 2y = 7$
$-x + 2y = 7$

21. $-18x + 6y = 24$
$3x - y = -2$

22. $4y + 5x = 15$
$x = 8y + 3$

23. $6x + 3y = 9$
$2x + 9y = 27$

24. ★ **MULTIPLE CHOICE** Which of the linear systems has *exactly* one solution?

Ⓐ $-x + y = 9$
$x - y = 9$

Ⓑ $-x + y = 9$
$x - y = -9$

Ⓒ $-x + y = 9$
$-x - y = 9$

Ⓓ $x - y = -9$
$-x + y = -9$

25. ★ **MULTIPLE CHOICE** Which of the linear systems has infinitely many solutions?

Ⓐ $15x + 5y = 20$
$6x - 2y = 8$

Ⓑ $15x - 5y = 20$
$6x - 2y = -8$

Ⓒ $15x - 5y = -20$
$6x - 2y = 8$

Ⓓ $15x - 5y = 20$
$6x - 2y = 8$

EXAMPLE 3
for Exs. 26–31

IDENTIFYING THE NUMBER OF SOLUTIONS **Without solving the linear system, tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.**

26. $y = -6x - 2$
$12x + 2y = -6$

27. $y = 7x + 13$
$-21x + 3y = 39$

28. $4x + 3y = 27$
$4x - 3y = -27$

29. $9x - 15y = 24$
$6x - 10y = 16$

30. $0.3x + 0.4y = 2.4$
$0.5x - 0.6y = 0.2$

31. $0.9x - 2.1y = 12.3$
$1.5x - 3.5y = 20.5$

32. ★ **OPEN-ENDED** Write a linear system so that it has infinitely many solutions, and one of the equations is $y = 3x + 2$.

33. ★ **OPEN-ENDED** Write a linear system so that it has no solution and one of the equations is $7x - 8y = -9$.

34. **REASONING** Give a counterexample for the following statement: If the graphs of the equations of a linear system have the same slope, then the linear system has no solution.

35. **CHALLENGE** Find values of p, q, and r that produce the solution(s).

a. No solution

b. Infinitely many solutions

c. One solution of (4, 1)

$px + qy = r$ **Equation 1**

$2x - 3y = 5$ **Equation 2**

PROBLEM SOLVING

EXAMPLE 4 for Exs. 36–38

36. **RECREATION** One admission to a roller skating rink costs x dollars and renting a pair of skates costs y dollars. A group pays \$243 for admission for 36 people and 21 skate rentals. Another group pays \$81 for admission for 12 people and 7 skate rentals. Is there enough information to determine the cost of one admission to the roller skating rink? *Explain.*

37. **TRANSPORTATION** A passenger train travels from New York City to Washington, D.C., then back to New York City. The table shows the number of coach tickets and business class tickets purchased for each leg of the trip. Is there enough information to determine the cost of one coach ticket? *Explain.*

Destination	Coach tickets	Business class tickets	Money collected (dollars)
Washington, D.C.	150	80	22,860
New York City	170	100	27,280

38. **PHOTOGRAPHY** In addition to taking pictures on your digital camera, you can record 30 second movies. All pictures use the same amount of memory, and all 30 second movies use the same amount of memory. The number of pictures and 30 second movies on 2 memory cards is shown.

a. Is there enough information given to determine the amount of memory used by a 30 second movie? *Explain.*

b. Given that a 30 second movie uses 50 times the amount of memory that a digital picture uses, can you determine the amount of memory used by a 30 second movie? *Explain.*

Size of card (megabytes)	64	256
Pictures	450	1800
Movies	7	28

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

39. **MULTI-STEP PROBLEM** Two people are training for a speed ice-climbing event. During a practice climb, one climber starts 15 seconds after the first climber. The rates that the climbers ascend are shown.

a. Let d be the distance (in feet) traveled by a climber t seconds after the first person starts climbing. Write a linear system that models the situation.

b. Graph the linear system from part (a). Does the second climber catch up to the first climber? *Explain.*

40. ★ **EXTENDED RESPONSE** Two employees at a banquet facility are given the task of folding napkins. One person starts folding napkins at a rate of 5 napkins per minute. The second person starts 10 minutes after the first person and folds napkins at a rate of 4 napkins per minute.

a. **Model** Let y be the number of napkins folded x minutes after the first person starts folding. Write a linear system that models the situation.

b. **Solve** Solve the linear system.

c. **Interpret** Does the solution of the linear system make sense in the context of the problem? *Explain.*

41. **CHALLENGE** An airplane has an average air speed of 160 miles per hour. The airplane takes 3 hours to travel with the wind from Salem to Lancaster. The airplane has to travel against the wind on the return trip. After 3 hours into the return trip, the airplane is 120 miles from Salem. Find the distance from Salem to Lancaster. If the problem cannot be solved with the information given, *explain* why.

Extension Use Piecewise Functions

GOAL Graph and write piecewise functions.

Key Vocabulary
- **piecewise function**
- **step function**

COMMON CORE

CC.9-12.F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*

A **piecewise function** is defined by at least two equations, each of which applies to a different part of the function's domain. An example is given below.

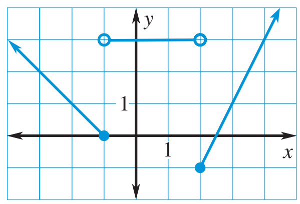
$$y = \begin{cases} x + 1, & \text{if } x < 0 \\ 2x - 1, & \text{if } x \geq 0 \end{cases}$$

The expression $x + 1$ gives the value of y when x is less than 0. The expression $2x - 1$ gives the value of y when x is greater than or equal to 0.

EXAMPLE 1 Graph a piecewise function

Graph the function: $y = \begin{cases} -x - 1, & \text{if } x \leq -1 \\ 3, & \text{if } -1 < x < 2 \\ 2x - 5, & \text{if } x \geq 2 \end{cases}$

Solution



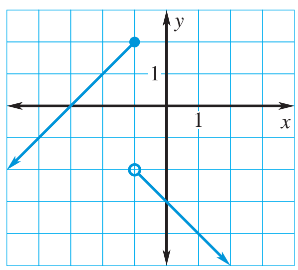
STEP 1 To the left of $x = -1$, graph $y = -x - 1$. Use a closed dot at $(-1, 0)$ because the equation applies when $x = -1$.

STEP 2 From $x = -1$ to $x = 2$, graph $y = 3$. Use open dots at $(-1, 3)$ and $(2, 3)$ because the equation does not apply when $x = -1$ or when $x = 2$.

STEP 3 To the right of $x = 2$, graph $y = 2x - 5$. Use a closed dot at $(2, -1)$ because the equation applies when $x = 2$.

EXAMPLE 2 Write a piecewise function

Write a piecewise function for the graph.



Solution

For $x \leq -1$, the graph is the line given by $y = x + 3$.

For $x > -1$, the graph is the line given by $y = -x - 3$.

▶ So, a piecewise function for the graph is as follows:

$$y = \begin{cases} x + 3, & \text{if } x \leq -1 \\ -x - 3, & \text{if } x > -1 \end{cases}$$

EXAMPLE 3 Solve a real-world problem

PARKING A parking garage charges \$5.00 for each hour or fraction of an hour up to 4 hours per day. Make a table of values. Then write the cost C (in dollars) as a piecewise function of the time t (in hours) parked and graph the function. What is the cost of parking for 2 hours and 9 minutes?

Solution

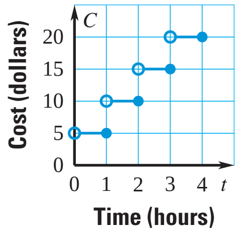
Table

Time (hours)	Cost (dollars)
$0 < t \le 1$	5.00
$1 < t \le 2$	10.00
$2 < t \le 3$	15.00
$3 < t \le 4$	20.00

Function rule

$$C = \begin{cases} 5, & \text{if } 0 < t \le 1 \\ 10, & \text{if } 1 < t \le 2 \\ 15, & \text{if } 2 < t \le 3 \\ 20, & \text{if } 3 < t \le 4 \end{cases}$$

Graph



▶ Because 2 hours and 9 minutes is between 2 and 3 hours, the cost is \$15.00.

STEP FUNCTIONS The function in Example 3 is called a *step function* because its graph resembles a set of stairs. A **step function** is a piecewise function that is defined by a constant value over each part of its domain.

PRACTICE

EXAMPLE 1
for Exs. 1–3

Graph the function.

1. $y = \begin{cases} x + 1, & \text{if } x < 0 \\ 0.5x - 1, & \text{if } x \ge 0 \end{cases}$

2. $y = \begin{cases} 2 + x, & \text{if } x < 0 \\ 2 - x, & \text{if } x \ge 0 \end{cases}$

3. $y = \begin{cases} 1, & \text{if } x < 0 \\ 2, & \text{if } 0 \le x < 1 \\ 3, & \text{if } x \ge 1 \end{cases}$

EXAMPLE 2
for Exs. 4–6

Write a piecewise function for the graph.

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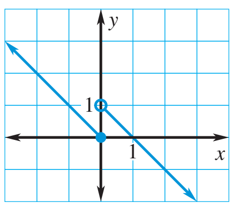

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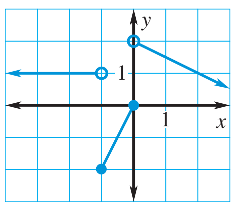

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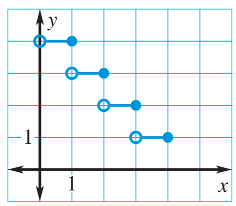

EXAMPLE 3
for Ex. 7

7. **PAY** Greg earns \$20 per hour when he works 40 or fewer hours in a week. When he works more than 40 hours in a week, he earns \$800 plus \$30 per hour for each hour over 40. Write a piecewise function that gives his weekly pay P for working t hours. Graph the function. What is his pay for 46 hours?

8. **REASONING** *Explain* why the parent absolute value function $y = |x|$ is a piecewise function. Write a piecewise rule for the function.

9. **REASONING** The output y of the *greatest integer function* is the greatest integer less than or equal to the input value x. Graph the function for $-4 \le x < 4$. Is it a piecewise function? a step function? *Explain.*