6.6 Solve Systems of Linear Inequalities

Before You graphed linear inequalities in two variables.

Now You will solve systems of linear inequalities in two variables.

Why So you can find a marching band's competition score, as in Ex. 36.

Key Vocabulary
- **system of linear inequalities**
- **solution of a system of linear inequalities**
- **graph of a system of linear inequalities**

A **system of linear inequalities** in two variables, or simply a *system of inequalities*, consists of two or more linear inequalities in the same variables. An example is shown.

$x - y > 7$ Inequality 1
$2x + y < 8$ Inequality 2

A **solution of a system of linear inequalities** is an ordered pair that is a solution of each inequality in the system. For example, $(6, -5)$ is a solution of the system above. The **graph of a system of linear inequalities** is the graph of all solutions of the system.

COMMON CORE

CC.9-12.A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

KEY CONCEPT *For Your Notebook*

Graphing a System of Linear Inequalities

STEP 1 **Graph** each inequality.

STEP 2 **Find** the intersection of the half-planes. The graph of the system is this intersection.

EXAMPLE 1 Graph a system of two linear inequalities

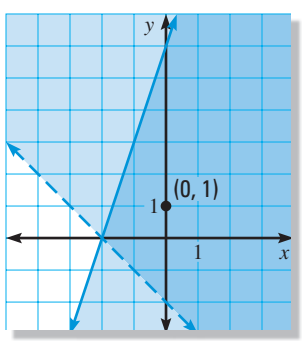
Graph the system of inequalities.

$y > -x - 2$ Inequality 1
$y \leq 3x + 6$ Inequality 2

REVIEW GRAPHING INEQUALITIES
You may want to review graphing a linear inequality in two variables before graphing systems of inequalities.

Solution

Graph both inequalities in the same coordinate plane. The graph of the system is the intersection of the two half-planes, which is shown as the darker shade of blue.



CHECK Choose a point in the dark blue region, such as $(0, 1)$. To check this solution, substitute 0 for x and 1 for y into each inequality.

$1 \stackrel{?}{>} 0 - 2$ | $1 \stackrel{?}{\leq} 0 + 6$

$1 > -2$ ✓ | $1 \leq 6$ ✓

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THE SOLUTION REGION In Example 1, the half-plane for each inequality is shaded, and the solution region is the intersection of the half-planes. From this point on, only the solution region will be shaded.

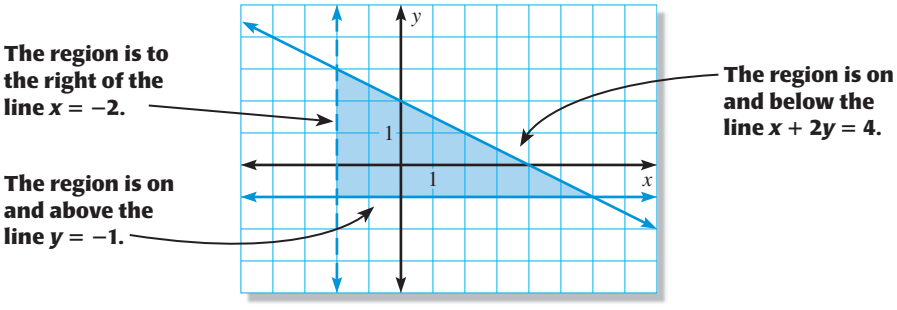
EXAMPLE 2 Graph a system of three linear inequalities

Graph the system of inequalities.

$y \geq -1$ **Inequality 1**
$x > -2$ **Inequality 2**
$x + 2y \leq 4$ **Inequality 3**

Solution

Graph all three inequalities in the same coordinate plane. The graph of the system is the triangular region shown.



GUIDED PRACTICE for Examples 1 and 2

Graph the system of linear inequalities.

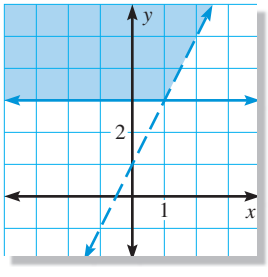
1. $y < x - 4$
$y \geq -x + 3$

2. $y \geq -x + 2$
$y < 4$
$x < 3$

3. $y > -x$
$y \geq x - 4$
$y < 5$

EXAMPLE 3 Write a system of linear inequalities

Write a system of inequalities for the shaded region.



REVIEW EQUATIONS OF LINES
You may want to review writing an equation of a line before writing a system of inequalities.

Solution

INEQUALITY 1: One boundary line for the shaded region is $y = 3$. Because the shaded region is *above* the *solid* line, the inequality is $y \geq 3$.

INEQUALITY 2: Another boundary line for the shaded region has a slope of 2 and a y-intercept of 1. So, its equation is $y = 2x + 1$. Because the shaded region is *above* the *dashed* line, the inequality is $y > 2x + 1$.

▶ The system of inequalities for the shaded region is:

$y \geq 3$ **Inequality 1**
$y > 2x + 1$ **Inequality 2**

EXAMPLE 4 Write and solve a system of linear inequalities

BASEBALL The National Collegiate Athletic Association (NCAA) regulates the lengths of aluminum baseball bats used by college baseball teams. The NCAA states that the length (in inches) of the bat minus the weight (in ounces) of the bat cannot exceed 3. Bats can be purchased at lengths from 26 to 34 inches.

a. Write and graph a system of linear inequalities that describes the information given above.

b. A sporting goods store sells an aluminum bat that is 31 inches long and weighs 25 ounces. Use the graph to determine if this bat can be used by a player on an NCAA team.

Solution

a. Let x be the length (in inches) of the bat, and let y be the weight (in ounces) of the bat. From the given information, you can write the following inequalities:

$x - y \leq 3$ **The difference of the bat's length and weight can be at most 3.**

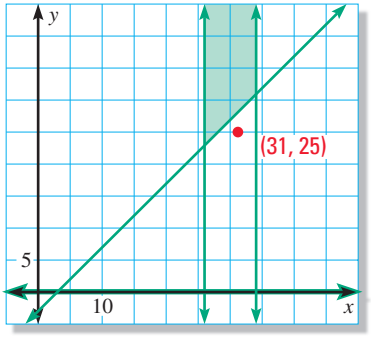
$x \geq 26$ **The length of the bat must be at least 26 inches.**

$x \leq 34$ **The length of the bat can be at most 34 inches.**

$y \geq 0$ **The weight of the bat cannot be a negative number.**

WRITING SYSTEMS OF INEQUALITIES
Consider the values of the variables when writing a system of inequalities. In many real-world problems, the values cannot be negative.

Graph each inequality in the system. Then identify the region that is common to all of the graphs of the inequalities. This region is shaded in the graph shown.



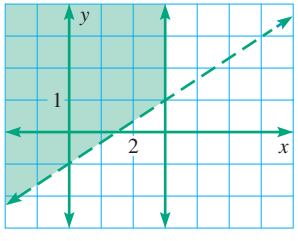
b. Graph the point that represents a bat that is 31 inches long and weighs 25 ounces.

▶ Because the point falls outside the solution region, the bat cannot be used by a player on an NCAA team.

GUIDED PRACTICE for Examples 3 and 4

Write a system of inequalities that defines the shaded region.

4.



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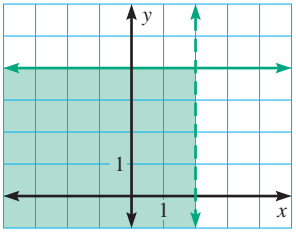


6. WHAT IF? In Example 4, suppose a Senior League (ages 10–14) player wants to buy the bat described in part (b). In Senior League, the length (in inches) of the bat minus the weight (in ounces) of the bat cannot exceed 8. Write and graph a system of inequalities to determine whether the described bat can be used by the Senior League player.

6.6 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 13 and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 22, 33, and 40

SKILL PRACTICE

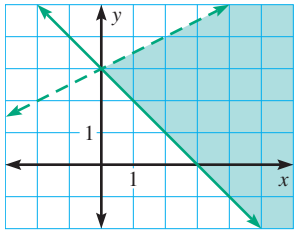
1. **VOCABULARY** Copy and complete: A(n) __?__ of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system.

2. ★ **WRITING** *Describe* the steps you would take to graph the system of inequalities shown.

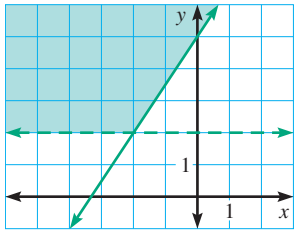
$x - y < 7$ **Inequality 1**
$y \geq 3$ **Inequality 2**

CHECKING A SOLUTION **Tell whether the ordered pair is a solution of the system of inequalities.**

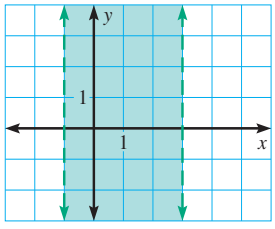
3. (1, 1)



4. (0, 6)



5. (3, −1)



EXAMPLE 1 for Exs. 6–17

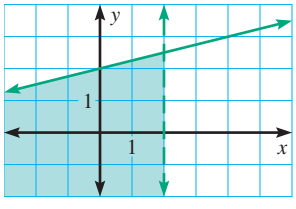
MATCHING SYSTEMS AND GRAPHS **Match the system of inequalities with its graph.**

6. $x - 4y > -8$
   $x \geq 2$

7. $x - 4y \geq -8$
   $x < 2$

8. $x - 4y > -8$
   $y \geq 2$

A.



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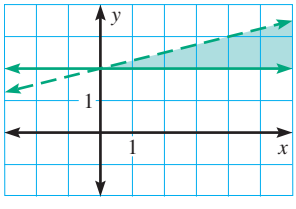

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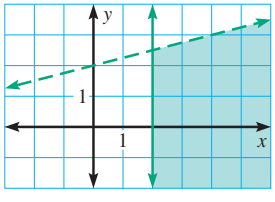


GRAPHING A SYSTEM **Graph the system of inequalities.**

9. $x > -5$
   $x < 2$

10. $y \leq 10$
    $y \geq 6$

11. $x > 3$
    $y > x$

12. $y < -2x + 3$
    $y \geq 4$

13. $y \geq 0$
    $y < 2.5x - 1$

14. $y \geq 2x + 1$
    $y < -x + 4$

15. $x < 8$
    $x - 4y \leq -8$

16. $y \geq -2$
    $2x + 3y > -6$

17. $y - 2x < 7$
    $y + 2x > -1$

EXAMPLE 2 for Exs. 18–21

18. $x < 4$
    $y > 1$
    $y \geq -x + 1$

19. $x \geq 0$
    $y \geq 0$
    $6x - y < 12$

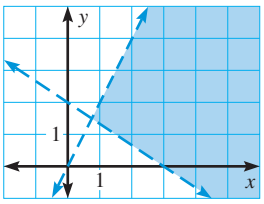
20. $x + y \leq 10$
    $x - y \geq 2$
    $y \geq 2$

21. ★ **MULTIPLE CHOICE** Which ordered pair is a solution of the system $2x - y \leq 5$ and $x + 2y > 2$?

Ⓐ (1, −1) Ⓑ (4, 1) Ⓒ (2, 0) Ⓓ (3, 2)

EXAMPLE 2 for Exs. 22–23

22. ★ **MULTIPLE CHOICE** The graph of which system of inequalities is shown?



Ⓐ $y < 2x$
$2x + 3y < 6$

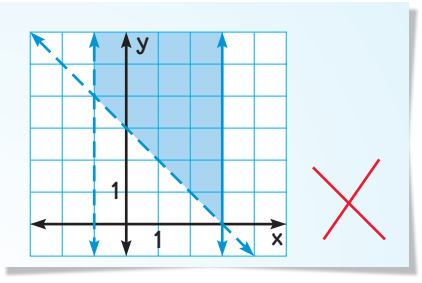
Ⓑ $y < 2x$
$2x + 3y > 6$

Ⓒ $y > 2x$
$2x + 3y < 6$

Ⓓ $y > 2x$
$2x + 3y > 6$

23. **ERROR ANALYSIS** *Describe* and correct the error in graphing this system of inequalities:

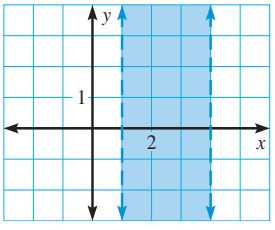
$x + y < 3$ **Inequality 1**
$x > -1$ **Inequality 2**
$x \leq 3$ **Inequality 3**



EXAMPLE 3 for Exs. 24–29

WRITING A SYSTEM **Write a system of inequalities for the shaded region.**

24.



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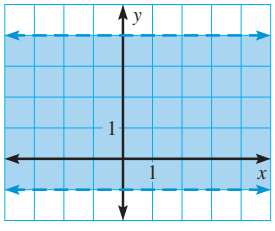

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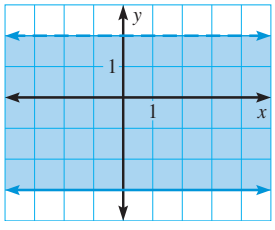

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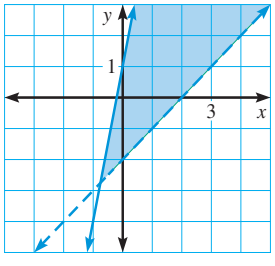

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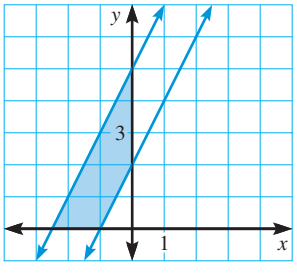

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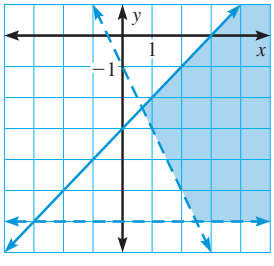

GRAPHING A SYSTEM **Graph the system of inequalities.**

30. $x > 4$
$x < 9$
$y \leq 2$
$y > -2$

31. $x + y < 4$
$x + y > -2$
$x - y \leq 3$
$x - y \geq -4$

32. $x \leq 10$
$3x + 2y \geq 9$
$x - 2y \leq 6$
$x + y \leq 5$

33. ★ **SHORT RESPONSE** Does the system of inequalities have any solutions? *Explain.*

$x - y > 5$ **Inequality 1**
$x - y < 1$ **Inequality 2**

CHALLENGE **Write a system of inequalities for the shaded region described.**

34. The shaded region is a rectangle with vertices at (2, 1), (2, 4), (6, 4), and (6, 1).

35. The shaded region is a triangle with vertices at (−3, 0), (3, 2), and (0, −2).

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

EXAMPLE 4 for Exs. 36–38

36. COMPETITION SCORES In a marching band competition, scoring is based on a musical evaluation and a visual evaluation. The musical evaluation score cannot exceed 60 points, the visual evaluation score cannot exceed 40 points. Write and graph a system of inequalities for the scores that a marching band can receive.

37. NUTRITION For a hiking trip, you are making a mix of x ounces of peanuts and y ounces of chocolate pieces. You want the mix to have less than 70 grams of fat and weigh less than 8 ounces. An ounce of peanuts has 14 grams of fat, and an ounce of chocolate pieces has 7 grams of fat. Write and graph a system of inequalities that models the situation.

38. FISHING LIMITS You are fishing in a marina for surfperch and rockfish, which are two species of bottomfish. Gaming laws in the marina allow you to catch no more than 15 surfperch per day, no more than 10 rockfish per day, and no more than 15 total bottomfish per day.

a. Write and graph a system of inequalities that models the situation.

b. Use the graph to determine whether you can catch 11 surfperch and 9 rockfish in one day.

39. HEALTH A person's maximum heart rate (in beats per minute) is given by $220 - x$ where x is the person's age in years ($20 \leq x \leq 65$). When exercising, a person should aim for a heart rate that is at least 70% of the maximum heart rate and at most 85% of the maximum heart rate.

a. Write and graph a system of inequalities that models the situation.

b. A 40-year-old person's heart rate varies from 104 to 120 beats per minute while exercising. Does his heart rate stay in the suggested target range for his age? *Explain.*

40. ★ SHORT RESPONSE A photography shop has a self-service photo center that allows you to make prints of pictures. Each sheet of printed pictures costs $8. The number of pictures that fit on each sheet is shown.

a. You want at least 16 pictures of any size, and you are willing to spend up to $48. Write and graph a system of inequalities that models the situation.

b. Will you be able to purchase 12 pictures that are 3 inches by 5 inches and 6 pictures that are 4 inches by 6 inches? *Explain.*

Four 3 inch by 5 inch pictures fit on one sheet.

Two 4 inch by 6 inch pictures fit on one sheet.

41. CHALLENGE You make necklaces and keychains to sell at a craft fair. The table shows the time that it takes to make each necklace and keychain, the cost of materials for each necklace and keychain, and the time and money that you can devote to making necklaces and keychains.

	Necklace	Keychain	Available
Time to make (hours)	0.5	0.25	20
Cost to make (dollars)	2	3	120

a. Write and graph a system of inequalities for the number x of necklaces and the number y of keychains that you can make under the given constraints.

b. Find the vertices (corner points) of the graph.

c. You sell each necklace for \$10 and each keychain for \$8. The revenue R is given by the equation $R = 10x + 8y$. Find the revenue for each ordered pair in part (b). Which vertex results in the maximum revenue?

QUIZ

Graph the linear system. Then use the graph to tell whether the linear system has *one solution*, *no solution*, or *infinitely many solutions*.

1. $x - y = 1$
$x - y = 6$

2. $6x + 2y = 16$
$2x - y = 2$

3. $3x - 3y = -2$
$-6x + 6y = 4$

Graph the system of linear inequalities.

4. $x > -3$
$x < 7$

5. $y \leq 2$
$y < 6x + 2$

6. $4x \geq y$
$-x + 4y < 4$

7. $x + y < 2$
$2x + y > -3$
$y \geq 0$

8. $y \geq 3x - 4$
$y \leq x$
$y \geq -5x - 15$

9. $x > -5$
$x < 0$
$y \leq 2x + 7$