

## REVIEW KEY VOCABULARY

- order of magnitude
- zero exponent
- negative exponent
- exponential function
- exponential growth
- growth factor, growth rate
- compound interest
- exponential decay
- decay factor, decay rate

## VOCABULARY EXERCISES

- Copy and complete: The function  $y = 1200(0.3)^t$  is an exponential ? function, and the base 0.3 is called the ?.
- WRITING** Explain how you can tell whether a table represents a linear function or an exponential function.

Tell whether the function represents exponential growth or exponential decay. *Explain.*

- $y = 3(0.85)^x$
- $y = \frac{1}{2}(1.01)^x$
- $y = 2(2.1)^x$

## REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

## 7.1

## Apply Exponent Properties Involving Products

## EXAMPLE

Simplify  $(3y^3)^4 \cdot y^5$ .

$$\begin{aligned} (3y^3)^4 \cdot y^5 &= 3^4 \cdot (y^3)^4 \cdot y^5 && \text{Power of a product property} \\ &= 81 \cdot y^{12} \cdot y^5 && \text{Power of a power property} \\ &= 81y^{17} && \text{Product of powers property} \end{aligned}$$

## EXERCISES

Simplify the expression.

- $4^4 \cdot 4^3$
- $(-3)^7(-3)$
- $z^3 \cdot z^5 \cdot z^5$
- $(y^4)^5$
- $[(-7)^4]^4$
- $[(b+2)^8]^3$
- $(6^4 \cdot 31)^5$
- $-(8xy)^2$
- $(2x^2)^4 \cdot x^5$
- EARTH SCIENCE** The order of magnitude of the mass of Earth's atmosphere is  $10^{18}$  kilograms. The order of magnitude of the mass of Earth's oceans is  $10^3$  times greater. What is the order of magnitude of the mass of Earth's oceans?

## EXAMPLES

1, 2, 3, 4,  
and 5

for Exs. 6–15

## 7.2

## Apply Exponent Properties Involving Quotients

## EXAMPLE

Simplify  $\left(\frac{x^3}{y}\right)^4 \cdot \frac{2}{x^5}$ .

$$\left(\frac{x^3}{y}\right)^4 \cdot \frac{2}{x^5} = \frac{(x^3)^4}{y^4} \cdot \frac{2}{x^5}$$

Power of a quotient property

$$= \frac{x^{12}}{y^4} \cdot \frac{2}{x^5}$$

Power of a power property

$$= \frac{2x^{12}}{y^4x^5}$$

Multiply fractions.

$$= \frac{2x^7}{y^4}$$

Quotient of powers property

## EXERCISES

Simplify the expression.

16.  $\frac{(-3)^7}{(-3)^3}$

17.  $\frac{5^2 \cdot 5^4}{5^3}$

18.  $\left(\frac{m}{n}\right)^3$

19.  $\frac{17^{12}}{17^8}$

20.  $\left(-\frac{1}{x}\right)^4$

21.  $\left(\frac{7x^5}{y^2}\right)^2$

22.  $\frac{1}{p^2} \cdot p^6$

23.  $\frac{6}{7r^{10}} \cdot \left(\frac{r^5}{s}\right)^5$

24. **PER CAPITA INCOME** The order of magnitude of the population of Montana in 2003 was  $10^6$  people. The order of magnitude of the total personal income (in dollars) for Montana in 2003 was  $10^{10}$ . What was the order of magnitude of the mean personal income in Montana in 2003?

EXAMPLES  
1, 2, and 3  
for Exs. 16–24

## 7.3

## Define and Use Zero and Negative Exponents

## EXAMPLE

Evaluate  $(2x^0y^{-5})^3$ .

$$(2x^0y^{-5})^3 = 2^3 \cdot x^0 \cdot y^{-15}$$

Power of a power property

$$= 8 \cdot 1 \cdot y^{-15}$$

Definition of zero exponent

$$= \frac{8}{y^{15}}$$

Definition of negative exponents

## EXERCISES

Evaluate the expression.

25.  $14^0$

26.  $3^{-4}$

27.  $\left(\frac{2}{3}\right)^{-3}$

28.  $7^{-5} \cdot 7^5$

29. **UNITS OF MEASURE** Use the fact that 1 femtogram =  $10^{-18}$  kilogram and 1 nanogram =  $10^{-12}$  kilogram to complete the following statement:  
1 nanogram =    femtogram(s).

EXAMPLES  
1, 2, and 4  
for Exs. 25–29

## 7.4 Write and Graph Exponential Growth Functions

### EXAMPLE

Graph the function  $y = 4^x$  and identify its domain and range.

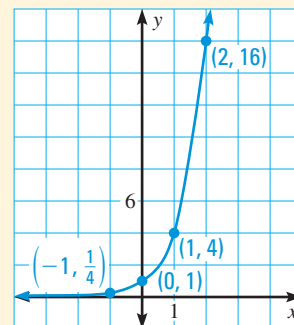
**STEP 1** Make a table. The domain is all real numbers.

$x$	-1	0	1	2
$y$	$\frac{1}{4}$	1	4	16

**STEP 2** Plot the points.

**STEP 3** Draw a smooth curve through the points.

**STEP 4** Identify the range. As you can see from the graph, the range is all positive real numbers.



### EXAMPLES

#### 2 and 3

for Exs. 30–34

### EXERCISES

Graph the function and identify its domain and range.

30.  $y = 6^x$

31.  $y = (1.1)^x$

32.  $y = (3.5)^x$

33.  $y = \left(\frac{5}{2}\right)^x$

34. Graph the function  $y = -5 \cdot 2^x$ . Compare the graph with the graph of  $y = 2^x$ .

## 7.5

## Write and Graph Exponential Decay Functions

## EXAMPLE 1

Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.

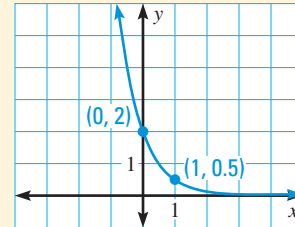
The graph represents exponential decay ( $y = ab^x$  where  $0 < b < 1$ ). The  $y$ -intercept is 2, so  $a = 2$ . Find the value of  $b$  by using the point (1, 0.5) and  $a = 2$ .

$$y = ab^x \quad \text{Write function.}$$

$$0.5 = 2 \cdot b^1 \quad \text{Substitute.}$$

$$0.25 = b \quad \text{Solve for } b.$$

A function rule is  $y = 2(0.25)^x$ .



## EXAMPLE 2

**CAR VALUE** A family purchases a car for \$11,000. The car depreciates (loses value) at a rate of about 16% annually. Write a function that models the value of the car over time. Find the approximate value of the car in 4 years.

Let  $V$  represent the value (in dollars) of the car, and let  $t$  represent the time (in years since the car was purchased). The initial value is 11,000, and the decay rate is 0.16.

$$V = a(1 - r)^t \quad \text{Write exponential decay model.}$$

$$= 11,000(1 - 0.16)^t \quad \text{Substitute 11,000 for } a \text{ and 0.16 for } r.$$

$$= 11,000(0.84)^t \quad \text{Simplify.}$$

To find the approximate value of the car in 4 years, substitute 4 for  $t$ .

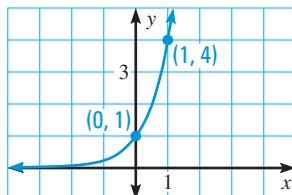
$$V = 11,000(0.84)^t = 11,000(0.84)^4 \approx \$5477$$

The approximate value of the car in 4 years is \$5477.

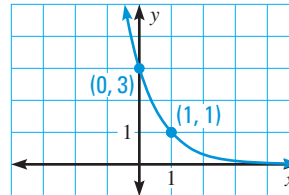
## EXERCISES

Tell whether the graph represents *exponential growth* or *exponential decay*. Then write a rule for the function.

35.



36.



37. **CAR VALUE** The value of a car is \$13,000. The car depreciates (loses value) at a rate of about 15% annually. Write an exponential decay model for the value of the car. Find the approximate value of the car in 4 years.

**EXAMPLES**  
4 and 5  
for Exs. 35–37

Simplify the expression. Write your answer using exponents.

1.  $(62 \cdot 17)^4$

2.  $(-3)(-3)^6$

3.  $\frac{8^4 \cdot 8^5}{8^3}$

4.  $(8^4)^3$

5.  $\frac{2^{15}}{2^8}$

6.  $5^3 \cdot 5^0 \cdot 5^5$

7.  $[(-4)^3]^2$

8.  $\frac{(-5)^{10}}{(-5)^3}$

Simplify the expression.

9.  $t^2 \cdot t^6$

10.  $\left(\frac{s}{t}\right)^6$

11.  $\frac{1}{9^{-2}}$

12.  $-(6p)^2$

13.  $(5xy)^2$

14.  $\frac{1}{z^7} \cdot z^9$

15.  $(x^5)^3$

16.  $\left(-\frac{4}{c}\right)^2$

Simplify the expression. Write your answer using only positive exponents.

17.  $\left(\frac{a^{-3}}{3b}\right)^4$

18.  $\frac{3}{4d} \cdot \frac{(2d)^4}{c^3}$

19.  $y^0 \cdot (8x^6y^{-3})^{-2}$

20.  $(5r^5)^3 \cdot r^{-2}$

21. Graph the function  $y = 4^x$ . Identify its domain and range.

22. Graph the function  $y = \frac{1}{2} \cdot 4^x$ . Compare the graph with the graph of  $y = 4^x$ .

23. **ANIMATION** About  $10^7$  bytes of data make up a single frame of an animated film. There are about  $10^3$  frames in 1 minute of a film. About how many bytes of data are there in 1 hour of an animated film?

24. **SALARY** A recent college graduate accepts a job at a law firm. The job has a salary of \$32,000 per year. The law firm guarantees an annual pay increase of 3% of the employee's salary.

- Write a function that models the employee's salary over time. Assume that the employee receives only the guaranteed pay increase.
- Use the function to find the employee's salary after 5 years.

25. **SCIENCE** At sea level, Earth's atmosphere exerts a pressure of 1 atmosphere. Atmospheric pressure  $P$  (in atmospheres) decreases with altitude and can be modeled by  $P = (0.99987)^a$  where  $a$  is the altitude (in meters).

- Identify the initial amount, decay factor, and decay rate.
- Use a graphing calculator to graph the function.
- Estimate the altitude at which the atmospheric pressure is about half of what it is at sea level.