# 7 Exponents and Exponential Functions



COMMON CORE

| Lesson | |
|---|---|
| 7.1 | CC.9-12.A.SSE.3c |
| 7.2 | CC.9-12.A.SSE.3c |
| 7.3 | CC.9-12.A.SSE.3c |
| 7.4 | CC.9-12.A.CED.2* |
| 7.5 | CC.9-12.A.CED.2* |

## Before

Previously, you learned the following skills, which you'll use in this chapter: using exponents, percents, and decimals, and writing function rules.

## Prerequisite Skills

### VOCABULARY CHECK

1. Identify the exponent and the base in the expression $13^8$.
2. Copy and complete: An expression that represents repeated multiplication of the same factor is called a(n) _?_.

### SKILLS CHECK

**Evaluate the expression.**

3. $x^2$ when $x = 10$ 4. $a^3$ when $a = 3$ 5. $r^2$ when $r = \frac{5}{6}$ 6. $z^3$ when $z = \frac{1}{2}$

**Order the numbers from least to greatest.**

7. 6.12, 6.2, 6.01 8. 0.073, 0.101, 0.0098

**Write the percent as a decimal.**

9. 4% 10. 0.5% 11. 13.8% 12. 145%

13. Write a rule for the function. Graph the function

| Input | 0 | 1 | 4 | 6 | 10 |
|---|---|---|---|---|---|
| Output | 2 | 3 | 6 | 8 | 12 |

## Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

## Big Ideas

1. **Applying properties of exponents to simplify expressions**
2. **Writing and graphing exponential functions**

### KEY VOCABULARY

- order of magnitude
- exponential function
- exponential growth
- compound interest
- exponential decay

## Why?

You can use exponents to explore exponential growth and decay. For example, you can write an exponential function to find the value of a collector car over time.

## Animated Algebra

The animation illustrated below helps you answer a question from this chapter: If you know the growth rate of the value of a collector car over time, can you predict what the car will sell for at an auction?


Animated Algebra at my.hrw.com

# Products and Powers

**MATERIALS** • paper and pencil

MATHEMATICAL PRACTICES

**Look for and make use of structure.**

**QUESTION** How can you find a product of powers and a power of a power?

## EXPLORE 1 Find products of powers

**STEP 1 *Copy and complete*** Copy and complete the table.

| Expression | Expression as repeated multiplication | Number of factors | Simplified expression |
|---|---|---|---|
| $7^4 \cdot 7^5$ | $(7 \cdot 7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)$ | 9 | $7^9$ |
| $(-4)^2 \cdot (-4)^3$ | $[(-4) \cdot (-4)] \cdot [(-4) \cdot (-4) \cdot (-4)]$ | ? | ? |
| $x^1 \cdot x^5$ | ? | ? | ? |

**STEP 2 *Analyze results*** Find a pattern that relates the exponents of the factors in the first column and the exponent of the expression in the last column.

## EXPLORE 2 Find powers of powers

**STEP 1 *Copy and complete*** Copy and complete the table.

| Expression | Expanded expression | Expression as repeated multiplication | Number of factors | Simplified expression |
|---|---|---|---|---|
| $(5^3)^2$ | $(5^3) \cdot (5^3)$ | $(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)$ | 6 | $5^6$ |
| $[(-6)^2]^4$ | $[(-6)^2] \cdot [(-6)^2] \cdot [(-6)^2] \cdot [(-6)^2]$ | ? | ? | ? |
| $(a^3)^3$ | ? | ? | ? | ? |

**STEP 2 *Analyze results*** Find a pattern that relates the exponents of the expression in the first column and the exponent of the expression in the last column.

## DRAW CONCLUSIONS Use your observations to complete these exercises

**Simplify the expression. Write your answer using exponents.**

**1.** $5^2 \cdot 5^3$ **2.** $(-6)^1 \cdot (-6)^4$ **3.** $m^6 \cdot m^4$

**4.** $(10^3)^3$ **5.** $[(-2)^3]^4$ **6.** $(c^2)^6$

**In Exercises 7 and 8, copy and complete the statement.**

**7.** If $a$ is a real number and $m$ and $n$ are positive integers, then $a^m \cdot a^n = \underline{\ ?\ }$.

**8.** If $a$ is a real number and $m$ and $n$ are positive integers, then $(a^m)^n = \underline{\ ?\ }$.

# 7.1 Apply Exponent Properties Involving Products

**Before** You evaluated exponential expressions.

**Now** You will use properties of exponents involving products.

**Why?** So you can evaluate agricultural data, as in Example 5.

**Key Vocabulary**
- order of magnitude
- power
- exponent
- base

Notice what happens when you multiply two powers that have the same base.

$$a^2 \cdot a^3 = \overbrace{\underbrace{(a \cdot a)}_{\text{2 factors}} \cdot \underbrace{(a \cdot a \cdot a)}_{\text{3 factors}}}^{\text{5 factors}} = a^5 = a^{2+3}$$

The example above suggests the following property of exponents, known as the product of powers property.

COMMON CORE

**CC.9-12.A.SSE.3c** Use the properties of exponents to transform expressions for exponential functions.

**KEY CONCEPT** *For Your Notebook*

**Product of Powers Property**

Let $a$ be a real number, and let $m$ and $n$ be positive integers.

**Words** To multiply powers having the same base, add the exponents.

**Algebra** $a^m \cdot a^n = a^{m+n}$ **Example** $5^6 \cdot 5^3 = 5^{6+3} = 5^9$

**EXAMPLE 1** Use the product of powers property

**SIMPLIFY EXPRESSIONS**
When simplifying powers with numerical bases only, write your answers using exponents, as in parts (a), (b), and (c).

**a.** $7^3 \cdot 7^5 = 7^{3+5} = 7^8$

**b.** $9 \cdot 9^8 \cdot 9^2 = 9^1 \cdot 9^8 \cdot 9^2$
$= 9^{1+8+2}$
$= 9^{11}$

**c.** $(-5)(-5)^6 = (-5)^1 \cdot (-5)^6$
$= (-5)^{1+6}$
$= (-5)^7$

**d.** $x^4 \cdot x^3 = x^{4+3} = x^7$

**GUIDED PRACTICE** for Example 1

**Simplify the expression.**

**1.** $3^2 \cdot 3^7$ **2.** $5 \cdot 5^9$ **3.** $(-7)^2(-7)$ **4.** $x^2 \cdot x^6 \cdot x$

**POWER OF A POWER** Notice what happens when you raise a power to a power.

$$(a^2)^3 = a^2 \cdot a^2 \cdot a^2 = (a \cdot a) \cdot (a \cdot a) \cdot (a \cdot a) = a^6 = a^{2 \cdot 3}$$

The example above suggests the following property of exponents, known as the power of a power property.

**KEY CONCEPT** *For Your Notebook*

### Power of a Power Property

Let $a$ be a real number, and let $m$ and $n$ be positive integers.

**Words** To find a power of a power, multiply exponents.

**Algebra** $(a^m)^n = a^{mn}$

**Example** $(3^4)^2 = 3^{4 \cdot 2} = 3^8$

## EXAMPLE 2 Use the power of a power property

**AVOID ERRORS**
In part (d), notice that you can write $[(y+2)^6]^2$ as $(y+2)^{12}$, but you cannot write $(y+2)^{12}$ as $y^{12} + 2^{12}$.

**a.** $(2^5)^3 = 2^{5 \cdot 3}$
$= 2^{15}$

**b.** $[(-6)^2]^5 = (-6)^{2 \cdot 5}$
$= (-6)^{10}$

**c.** $(x^2)^4 = x^{2 \cdot 4}$
$= x^8$

**d.** $[(y+2)^6]^2 = (y+2)^{6 \cdot 2}$
$= (y+2)^{12}$

**GUIDED PRACTICE** for Example 2

**Simplify the expression.**

**5.** $(4^2)^7$ **6.** $[(-2)^4]^5$ **7.** $(n^3)^6$ **8.** $[(m+1)^5]^4$

**POWER OF A PRODUCT** Notice what happens when you raise a product to a power.

$$(ab)^3 = (ab) \cdot (ab) \cdot (ab) = (a \cdot a \cdot a) \cdot (b \cdot b \cdot b) = a^3b^3$$

The example above suggests the following property of exponents, known as the power of a product property.

**KEY CONCEPT** *For Your Notebook*

### Power of a Product Property

Let $a$ and $b$ be real numbers, and let $m$ be a positive integer.

**Words** To find a power of a product, find the power of each factor and multiply.

**Algebra** $(ab)^m = a^mb^m$

**Example** $(23 \cdot 17)^5 = 23^5 \cdot 17^5$

## EXAMPLE 3 Use the power of a product property

**SIMPLIFY EXPRESSIONS**
When simplifying powers with numerical *and* variable bases, be sure to evaluate the numerical power, as in parts (b), (c), and (d).

**a.** $(24 \cdot 13)^8 = 24^8 \cdot 13^8$

**b.** $(9xy)^2 = (9 \cdot x \cdot y)^2 = 9^2 \cdot x^2 \cdot y^2 = 81x^2y^2$

**c.** $(-4z)^2 = (-4 \cdot z)^2 = (-4)^2 \cdot z^2 = 16z^2$

**d.** $-(4z)^2 = -(4 \cdot z)^2 = -(4^2 \cdot z^2) = -16z^2$

## EXAMPLE 4 Use all three properties

**Simplify $(2x^3)^2 \cdot x^4$.**

$(2x^3)^2 \cdot x^4 = 2^2 \cdot (x^3)^2 \cdot x^4$ **Power of a product property**

$= 4 \cdot x^6 \cdot x^4$ **Power of a power property**

$= 4x^{10}$ **Product of powers property**

Animated Algebra at my.hrw.com

**ORDER OF MAGNITUDE** The **order of magnitude** of a quantity can be defined as the power of 10 nearest the quantity. Order of magnitude can be used to estimate or perform rough calculations. For instance, there are about 91,000 species of insects in the United States. The power of 10 closest to 91,000 is $10^5$, or 100,000. So, there are about $10^5$ species of insects in the United States.

## EXAMPLE 5 Solve a real-world problem

**BEES** In 2003 the U.S. Department of Agriculture (USDA) collected data on about $10^3$ honeybee colonies. There are about $10^4$ bees in an average colony during honey production season. About how many bees were in the USDA study?

### Solution

To find the total number of bees, find the product of the number of colonies, $10^3$, and the number of bees per colony, $10^4$.

$10^3 \cdot 10^4 = 10^{3+4} = 10^7$

▶ The USDA studied about $10^7$, or 10,000,000, bees.

## ✓ GUIDED PRACTICE for Examples 3, 4, and 5

**Simplify the expression.**

**9.** $(42 \cdot 12)^2$ **10.** $(-3n)^2$ **11.** $(9m^3n)^4$ **12.** $5 \cdot (5x^2)^4$

**13.** **WHAT IF?** In Example 5, $10^2$ honeybee colonies in the study were located in Idaho. About how many bees were studied in Idaho?

# 7.1 EXERCISES

**HOMEWORK KEY**

○ = See **WORKED-OUT SOLUTIONS** Exs. 31 and 55

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 40, 41, 50, and 58

◆ = **MULTIPLE REPRESENTATIONS** Ex. 55

## SKILL PRACTICE

**1.** **VOCABULARY** Copy and complete: The __?__ of the quantity 93,534,004 people is the power of 10 nearest the quantity, or $10^8$ people.

**2.** ★ **WRITING** *Explain* when and how to use the product of powers property.

**EXAMPLES 1,2,3, and 4** for Exs. 3–41

**SIMPLIFYING EXPRESSIONS** **Simplify the expression. Write your answer using exponents.**

**3.** $4^2 \cdot 4^6$ **4.** $8^5 \cdot 8^2$ **5.** $3^3 \cdot 3$ **6.** $9 \cdot 9^5$

**7.** $(-7)^4(-7)^5$ **8.** $(-6)^6(-6)$ **9.** $2^4 \cdot 2^9 \cdot 2$ **10.** $(-3)^2(-3)^{11}(-3)$

**11.** $(3^5)^2$ **12.** $(7^4)^3$ **13.** $[(-5)^3]^4$ **14.** $[(-8)^9]^2$

**15.** $(15 \cdot 29)^3$ **16.** $(17 \cdot 16)^4$ **17.** $(132 \cdot 9)^6$ **18.** $((-14) \cdot 22)^5$

**SIMPLIFYING EXPRESSIONS** **Simplify the expression.**

**19.** $x^4 \cdot x^2$ **20.** $y^9 \cdot y$ **21.** $z^2 \cdot z \cdot z^3$ **22.** $a^4 \cdot a^3 \cdot a^{10}$

**23.** $(x^5)^2$ **24.** $(y^4)^6$ **25.** $[(b-2)^2]^6$ **26.** $[(d+9)^7]^3$

**27.** $(-5x)^2$ **28.** $-(5x)^2$ **29.** $(7xy)^2$ **30.** $(5pq)^3$

**31.** $(-10x^6)^2 \cdot x^2$ **32.** $(-8m^4)^2 \cdot m^3$ **33.** $6d^2 \cdot (2d^5)^4$ **34.** $(-20x^3)^2(-x^7)$

**35.** $-(2p^4)^3(-1.5p^7)$ **36.** $\left(\frac{1}{2}y^5\right)^3(2y^2)^4$ **37.** $(3x^5)^3(2x^7)^2$ **38.** $(-10n)^2(-4n^3)^3$

**39.** **ERROR ANALYSIS** *Describe* and correct the error in simplifying $c \cdot c^4 \cdot c^5$.

$$\begin{aligned} c \cdot c^4 \cdot c^5 &= c^1 \cdot c^4 \cdot c^5 \\ &= c^{1 \cdot 4 \cdot 5} \\ &= c^{20} \end{aligned}$$

**40.** ★ **MULTIPLE CHOICE** Which expression is equivalent to $(-9)^6$?

Ⓐ $(-9)^2(-9)^3$ Ⓑ $(-9)(-9)^5$ Ⓒ $[(-9)^4]^2$ Ⓓ $[(-9)^3]^3$

**41.** ★ **MULTIPLE CHOICE** Which expression is equivalent to $36x^{12}$?

Ⓐ $(6x^3)^4$ Ⓑ $12x^4 \cdot 3x^3$ Ⓒ $3x^3 \cdot (4x^3)^3$ Ⓓ $(6x^5)^2 \cdot x^2$

**SIMPLIFYING EXPRESSIONS** **Find the missing exponent.**

**42.** $x^4 \cdot x^? = x^5$ **43.** $(y^8)^? = y^{16}$ **44.** $(2z^?)^3 = 8z^{15}$ **45.** $(3a^3)^? \cdot 2a^3 = 18a^9$

**46.** **POPULATION** The population of New York City in 2000 was 8,008,278. What was the order of magnitude of the population of New York City?

**SIMPLIFYING EXPRESSIONS** **Simplify the expression.**

**47.** $(-3x^2y)^3(11x^3y^5)^2$ **48.** $-(-xy^2z^3)^5(x^4yz)^2$ **49.** $(-2s)(-5r^3st)^3(-2r^4st^7)^2$

**50.** ★ **OPEN-ENDED** Write three expressions involving products of powers, powers of powers, or powers of products that are equivalent to $12x^8$.

**51.** **CHALLENGE** Show that when $a$ and $b$ are real numbers and $n$ is a positive integer, $(ab)^n = a^n b^n$.

## PROBLEM SOLVING

**EXAMPLE 5** for Exs. 52–56

**52.** **ICE CREAM COMPOSITION** There are about 954,930 air bubbles in 1 cubic centimeter of ice cream. There are about 946 cubic centimeters in 1 quart. Use order of magnitude to find the approximate number of air bubbles in 1 quart of ice cream.

**53.** **ASTRONOMY** The order of magnitude of the radius of our solar system is $10^{13}$ meters. The order of magnitude of the radius of the visible universe is $10^{13}$ times as great. Find the approximate radius of the visible universe.

**54.** **COASTAL LANDSLIDE** There are about 1 billion grains of sand in 1 cubic foot of sand. In 1995 a stretch of beach at Sleeping Bear Dunes National Lakeshore in Michigan slid into Lake Michigan. Scientists believe that around 35 million cubic feet of sand fell into the lake. Use order of magnitude to find about how many grains of sand slid into the lake.

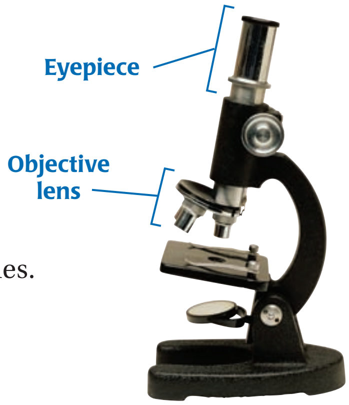
**55.** **MULTIPLE REPRESENTATIONS** There are about $10^{23}$ atoms of gold in 1 ounce of gold.

**a. Making a Table** Copy and complete the table by finding the number of atoms of gold for the given amounts of gold (in ounces).

| Gold (ounces) | 10 | 100 | 1000 | 10,000 | 100,000 |
|---|---|---|---|---|---|
| Number of atoms | ? | ? | ? | ? | ? |

**b. Writing an Expression** A particular mine in California extracted about 96,000 ounces of gold in 1 year. Use order of magnitude to write an expression you can use to find the approximate number of atoms of gold extracted in the mine that year. Simplify the expression. Verify your answer using the table.

**56.** **MULTI-STEP PROBLEM** A microscope has two lenses, the objective lens and the eyepiece, that work together to magnify an object. The total magnification of the microscope is the product of the magnification of the objective lens and the magnification of the eyepiece.

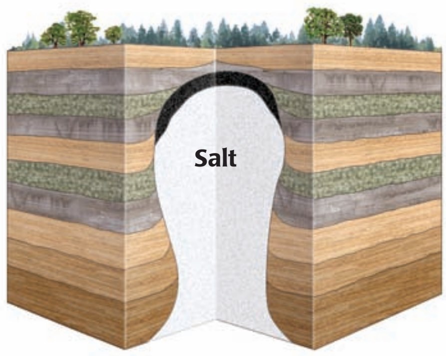



**a.** Your microscope's objective lens magnifies an object $10^2$ times, and the eyepiece magnifies an object 10 times. What is the total magnification of your microscope?

**b.** You magnify an object that is $10^2$ nanometers long. How long is the magnified image?

**57. VOLUME OF THE SUN** The radius of the sun is about 695,000,000 meters. The formula for the volume of a sphere, such as the sun, is $V = \frac{4}{3}\pi r^3$. Because the order of magnitude of $\frac{4}{3}\pi$ is 1, it does not contribute to the formula in a significant way. So, you can find the order of magnitude of the volume of the sun by cubing its radius. Find the order of magnitude of the volume of the sun.

**58. ★ EXTENDED RESPONSE** Rock salt can be mined from large deposits of salt called salt domes. A particular salt dome is roughly cylindrical in shape. The order of magnitude of the radius of the salt dome is $10^3$ feet. The order of magnitude of the height of the salt dome is about 10 times that of its radius. The formula for the volume of a cylinder is $V = \pi r^2 h$.




**a. Calculate** What is the order of magnitude of the height of the salt dome?

**b. Calculate** What is the order of magnitude of the volume of the salt dome?

**c. Explain** The order of magnitude of the radius of a salt dome can be 10 times the radius of the salt dome described in this exercise. What effect does multiplying the order of magnitude of the radius of the salt dome by 10 have on the volume of the salt dome? *Explain.*

**59. CHALLENGE** Your school is conducting a poll that has two parts, one part that has 13 questions and a second part that has 10 questions. Students can answer the questions in either part with “agree” or “disagree.” What power of 2 represents the number of ways there are to answer the questions in the first part of the poll? What power of 2 represents the number of ways there are to answer the questions in the second part of the poll? What power of 2 represents the number of ways there are to answer all of the questions on the poll?