Extension

Define and Use Fractional Exponents

GOAL Use fractional exponents.

Key Vocabulary
- cube root

COMMON CORE

CC.9-12.N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

You have learned to write the square root of a number using a radical sign. You can also write a square root of a number using exponents.

For any $a \geq 0$, suppose you want to write $\sqrt{a}$ as a^k. Recall that a number b (in this case, a^k) is a square root of a number a provided $b^2 = a$. Use this definition to find a value for k as follows.

$$b^2 = a \qquad \text{Definition of square root}$$

$$(a^k)^2 = a \qquad \text{Substitute } a^k \text{ for } b.$$

$$a^{2k} = a^1 \qquad \text{Power of a power property}$$

Because the bases are the same in the equation $a^{2k} = a^1$, the exponents must be equal:

$$2k = 1 \qquad \text{Set exponents equal.}$$

$$k = \frac{1}{2} \qquad \text{Solve for } k.$$

So, for a nonnegative number a, $\sqrt{a} = a^{1/2}$.

You can work with exponents of $\frac{1}{2}$ and multiples of $\frac{1}{2}$ just as you work with integer exponents.

EXAMPLE 1 Evaluate expressions involving square roots

a. $16^{1/2} = \sqrt{16}$
$= 4$

b. $25^{-1/2} = \frac{1}{25^{1/2}}$
$= \frac{1}{\sqrt{25}}$
$= \frac{1}{5}$

c. $9^{5/2} = 9^{(1/2) \cdot 5}$
$= (9^{1/2})^5$
$= (\sqrt{9})^5$
$= 3^5$
$= 243$

d. $4^{-3/2} = 4^{(1/2) \cdot (-3)}$
$= (4^{1/2})^{-3}$
$= (\sqrt{4})^{-3}$
$= 2^{-3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

FRACTIONAL EXPONENTS You can work with other fractional exponents just as you did with $\frac{1}{2}$.

CUBE ROOTS If $b^3 = a$, then b is the **cube root** of a. For example, $2^3 = 8$, so 2 is the cube root of 8. The cube root of a can be written as $\sqrt[3]{a}$ or $a^{1/3}$.

EXAMPLE 2 Evaluate expressions involving cube roots

a. $27^{1/3} = \sqrt[3]{27} = \sqrt[3]{3^3} = 3$

b. $8^{-1/3} = \dfrac{1}{8^{1/3}} = \dfrac{1}{\sqrt[3]{8}} = \dfrac{1}{2}$

c. $64^{4/3} = 64^{(1/3)\cdot 4} = (64^{1/3})^4 = (\sqrt[3]{64})^4 = 4^4 = 256$

d. $125^{-2/3} = 125^{(1/3)\cdot(-2)} = (125^{1/3})^{-2} = (\sqrt[3]{125})^{-2} = 5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

PROPERTIES OF EXPONENTS The properties of exponents for integer exponents also apply to fractional exponents.

EXAMPLE 3 Use properties of exponents

a. $12^{-1/2} \cdot 12^{5/2} = 12^{(-1/2)+(5/2)} = 12^{4/2} = 12^2 = 144$

b. $\dfrac{6^{4/3}\cdot 6}{6^{1/3}} = \dfrac{6^{(4/3)+1}}{6^{1/3}} = \dfrac{6^{7/3}}{6^{1/3}} = 6^{(7/3)-(1/3)} = 6^2 = 36$

PRACTICE

EXAMPLES 1, 2, and 3 for Exs. 1–12

Evaluate the expression.

1. $100^{3/2}$

2. $121^{-1/2}$

3. $81^{-3/2}$

4. $216^{2/3}$

5. $27^{-1/3}$

6. $343^{-2/3}$

7. $9^{7/2} \cdot 9^{-3/2}$

8. $\left(\frac{1}{16}\right)^{1/2}\left(\frac{1}{16}\right)^{-1/2}$

9. $36^{5/2} \cdot \dfrac{36^{-1/2}}{(36^{-1})^{-7/2}}$

10. $(27^{-1/3})^3$

11. $(-64)^{-5/3}(-64)^{4/3}$

12. $(-8)^{1/3}(-8)^{-2/3}(-8)^{1/3}$

13. REASONING Let $x > 0$. Compare the values of $x^{1/2}$ and $x^{-1/2}$. Give examples to support your thinking.