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CHAPTER 7 Exponential Functions: Continuous or Noncontinuous

Many of the functions you have seen so far are continuous. A *continuous* function is defined over all real numbers. Its graph is a smooth curve with no holes, breaks, jumps, or sharp turns. If a function is not defined over all real numbers, or its graph has holes, breaks, jumps, or sharp turns, then it is a *noncontinuous* function.

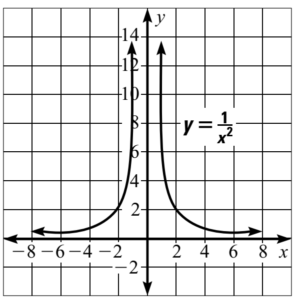
EXAMPLE 1 Identify continuous and noncontinuous functions

Graph the functions and explain whether they are continous or noncontinuous.

a. $y = \frac{1}{x^2}$ **b.** $y = \begin{cases} 2x, \text{ when } x > 2 \\ x^2, \text{ when } x < 2 \end{cases}$ **c.** $y = 2^x$

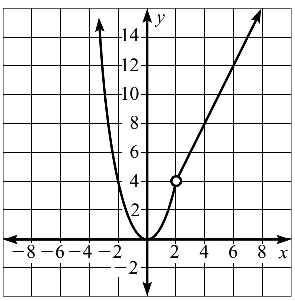
Solution:

a.



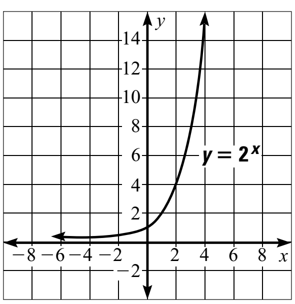
The function $y = \frac{1}{x^2}$ is noncontinuous since there is a break in the graph at $x = 0$. This is because the function is undefined at $x = 0$.

b.



The function $y = \begin{cases} 2x, \text{ when } x > 2 \\ x^2, \text{ when } x < 2 \end{cases}$ is noncontinuous since there is a hole in the graph at $x = 2$. This is because the function is not defined at $x = 2$.

c.



The function $y = 2^x$ is continuous since there are no breaks or holes anywhere in the graph. This is because the function is defined for all real numbers. ■

In Example 1, part c shows a continuous exponential function, $y = 2^x$. Other exponential functions and their graphs are explored in Example 2 to determine whether they are continuous or noncontinuous.

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Exponential Functions: Continuous or Noncontinuous *continued*

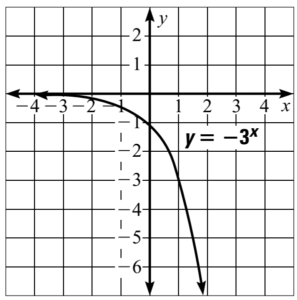
EXAMPLE 2 Graph exponential functions

Graph the exponential functions and determine whether they are continuous or noncontinuous.

a. $y = -3^x$ **b.** $y = 4^{(x-2)}$ **c.** $y = 2 \cdot \left(\frac{1}{4}\right)^x$

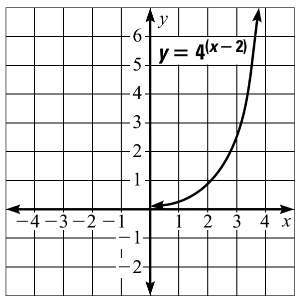
Solution:

a.



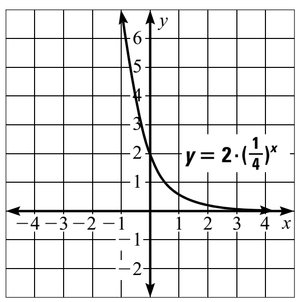
The graph of this exponential function is continuous.

b.



The graph of this exponential function is continuous.

c.



The graph of this exponential function is continuous. ■

Notice that each graph in Example 2 is continuous since each exponential function is defined for all real numbers. An exponential function will always be continuous unless there are values for which the function is undefined.

Examples 3 and 4 explore irrational exponents to determine if an exponential function could have any undefined values.

EXAMPLE 3 Irrational exponents

Use a graphing calculator to give an approximate value of the expression. Determine if the result is a real number.

a. 2^{π} **b.** $3^{\sqrt{2}}$ **c.** $2^{\sqrt{3}}$ **d.** $\pi^{\sqrt{2}}$

Solution:

a. $2^{\pi} \approx 8.825$ **b.** $3^{\sqrt{2}} \approx 4.7288$ **c.** $2^{\sqrt{3}} \approx 3.322$ **d.** $\pi^{\sqrt{2}} \approx 5.0475$

Each result is an irrational number. Therefore, each result is a real number. ■

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Consider the exponential function $y = 2^x$ from Example 1, part c. We saw that the graph of this function is continuous since there are no breaks or holes in its graph. So, $y = 2^x$ is defined for any real number. Suppose the value of the exponent is an irrational number such as π or $\sqrt{3}$. We know that $y = 2^{\pi}$ and $y = 2^{\sqrt{3}}$ must be defined.

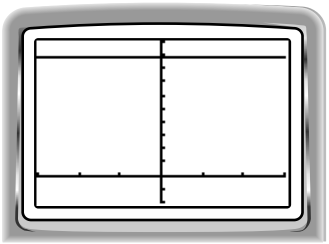
EXAMPLE 4

Exponential functions with irrational exponents

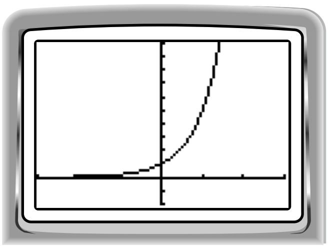
Use a graphing calculator to graph the exponential functions. Determine whether the graphs are continuous or noncontinuous.

a. $y = 2^{\pi}$ **b.** $y = \pi^{(\sqrt{2})x}$

Solution:

a. 

This graph is a straight line, so it is continuous.

b. 

The graph of this exponential function with an irrational number in the exponent is continuous. ■

Exponential functions are defined for all rational and irrational numbers, so they are defined for all real numbers. As a result, their graphs are continuous.

Practice

1. **Writing** Explain the difference between graphs of a continuous and a noncontinuous function.

2. **Writing** Explain whether or not all exponential functions are continuous.

Sketch a graph of the function. Determine whether the graph is continuous or noncontinuous.

3. $y = \begin{cases} \sqrt{x}, \text{ when } x > 0 \\ -2x, \text{ when } x < 0 \end{cases}$

4. $y = \begin{cases} \frac{1}{2}x^2, \text{ when } x \le 2 \\ x, \text{ when } x > 2 \end{cases}$

5. $y = \frac{2}{x}$

6. $y = -|x|$

7. $y = x^4$

8. $y = 4^x$

9. $y = 2 \cdot 3^x$

10. $y = -2^{(\pi x)}$

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Exponential Functions: Continuous or Noncontinuous *continued*

Problem Solving

11. Michaela bought a new car for \$25,000. The value of the car depreciates 15% each year. After t years, the value of the car, v, is given by the function $v = 25{,}000(0.85)^t$. Find the value of the car to the nearest dollar after 1, 2, and 3 years. Then graph this function and determine whether it is continuous or noncontinuous.

12. Mr. Gordon increases the price of each bicycle in his bicycle shop by 5% each year. A bicycle that costs \$200 this year will cost $200(1.05)^y$ y years from now. Explain whether this function is continuous or noncontinuous.

13. The population of a city is expected to increase by 0.8% each year. The current population is approximately 16 million. After t years, the population of this city, in millions, is expected to be $p = 16(1.008)^t$. Use a graphing calculator to graph this function. Explain whether it is continuous or noncontinuous.

14. Alex bought a computer for school last year. The value of the computer n years from now is modeled by the function $v = 750 \cdot 0.875^{(n+1)}$. Use a graphing calculator to graph this function. Explain whether the graph of this function is continuous or noncontinuous. Determine whether or not the value of Alex's computer ever equals \$0.

15. An environmentalist studying a certain endangered species counts 120 of these animals in their habitat. The population, p, of this species is decreasing according to the model $p = 120(0.96)^y$, where y is the number of years from now. About how many of these animals are expected to remain in their habitat 10 years from now? Graph this function.

16. The monthly sales of cell phones have increased at one retail shop according to the model $s = 250(1.35)^m$, where m is the number of months from now. Explain whether this function is continuous or noncontinuous.

17. The half-life of a substance is the amount of time it takes half of the substance to decay or decompose. A chemist determined the half-life of a radioactive substance to be 18 hours. The function $a = 500 \cdot \left(\frac{1}{2}\right)^{h/18}$ models the amount left, a, of 500 milligrams of this substance h hours from now. Use a graphing calculator to graph this function. Determine whether the graph of this function is continuous or noncontinuous.