

CHAPTER
7

Average Rates of Change

KEY CONCEPT

Average Rate of Change

A function's *average rate of change* is the amount the function increases or decreases over an interval. For a linear function, slope is a measure of the average rate of change.

EXAMPLE 1 Average rate of change in a linear function

Find and compare the average rates of change in each interval for the linear function $y = 4x$.

a. $0 \leq x \leq 1$

b. $1 \leq x \leq 2$

c. $0 \leq x \leq 2$

Solution:

	Interval	Endpoints	Average Rate of Change
a.	$0 \leq x \leq 1$	(0, 0) and (1, 4)	$\frac{4 - 0}{1 - 0} = 4$
b.	$1 \leq x \leq 2$	(1, 4) and (2, 8)	$\frac{8 - 4}{2 - 1} = 4$
c.	$0 \leq x \leq 2$	(0, 0) and (2, 8)	$\frac{8 - 0}{2 - 0} = 4$

The average rates of change are the same in each interval. Since the function is linear, the average rate of change is constant. ■

EXAMPLE 2 Average rate of change in an exponential function

Find and compare the average rates of change in each interval for the exponential function $y = 4^x$.

a. $0 \leq x \leq 1$

b. $1 \leq x \leq 2$

c. $0 \leq x \leq 2$

Solution:

	Interval	Endpoints	Average Rate of Change
a.	$0 \leq x \leq 1$	(0, 1) and (1, 4)	$\frac{4 - 1}{1 - 0} = 3$
b.	$1 \leq x \leq 2$	(1, 4) and (2, 16)	$\frac{16 - 4}{2 - 1} = 12$
c.	$0 \leq x \leq 2$	(0, 1) and (2, 16)	$\frac{16 - 1}{2 - 0} = \frac{15}{2} = 7.5$

The average rates of change are all different. The average rate of change is not constant. ■

Since the slope of a linear function is constant; its average rate of change is the same over all intervals. For an exponential function; its average rate of change is not constant and depends on the interval.

Average Rates of Change *continued***EXAMPLE 3** **Average rates of change in other non-linear functions**

Describe each function's average rate of change by finding the average rate of change over two intervals.

a. quadratic function: $y = x^2 - 3$

b. cubic function: $y = 2x^3 + 1$

Solution:

a.	Interval	Endpoints	Average Rate of Change
	$0 \leq x \leq 1$	$(0, -3)$ and $(1, -2)$	$\frac{-2 - (-3)}{1 - 0} = 1$
	$1 \leq x \leq 2$	$(1, -2)$ and $(2, 1)$	$\frac{1 - (-2)}{2 - 1} = 3$

The two average rates of change are different, so the average rate of change varies in a quadratic function.

b.	Interval	Endpoints	Average Rate of Change
	$1 \leq x \leq 2$	$(1, 3)$ and $(2, 17)$	$\frac{17 - 3}{2 - 1} = 14$
	$2 \leq x \leq 3$	$(2, 17)$ and $(3, 55)$	$\frac{55 - 17}{3 - 2} = 38$

The two average rates of change are different, so the average rate of change varies in a cubic function. ■

Practice

Describe the average rate of change of the function.

Explain your reasoning.

1. $y = -2^x + 2$

2. $y = \frac{x+1}{2}$

3. $y = x^2 + x - 6$

4. $y = \sqrt{x}$

Problem Solving

- Write a function that has a constant rate of change. Explain how you know it has a constant rate of change.
- Write a function that does not have a constant rate of change. Explain how you know the rate of change is not constant.
- Find the average rate of change of the function $y = x^3$ over the intervals $-1 \leq x \leq 0$, $0 \leq x \leq 1$, and $-1 \leq x \leq 1$. Explain whether or not it can be concluded that the average rate of change is constant for this function.
- The function below has a constant rate of change.

x	2	4	6	8	...
y	-7	n	-1	2	...

What is the value of n ?

- A coin in Amber's collection increases in value 20% each year. Last year, the coin was worth \$2.00. What is the value of the coin this year? What is the expected value of the coin 5 years from now?