Name ______________________ Date ______________

CHAPTER 7

Model Limitations

Many exponential models are valid for predicting the rate of growth or decay over a short period of time. However, these models are often limited when it comes to making predictions farther into the future or when making conclusions about occurrences too far in the past.

EXAMPLE

Limitations of a population model

During the 1990s, the population of a certain city increased from 80,000 to 180,000. The chart below lists the population of the city by year.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Population (thousands)	80	88	95	110	129	131	140	152	167	180

This data is approximated by the exponential growth model $P = 80(2.5)^{0.1t}$, where t is the number of years since 1990.

a. The actual population of this city in the year 2005 is approximately 301,000. Explain whether or not this model is a good predictor of this population.

b. Use the growth model to predict the population of this city in the year 100 years after 1990. Explain whether you believe this prediction could be accurate.

c. Use the growth model to calculate the population of this city in the year 1989 and then in 1980. What conclusions can be made about this population model based on these results?

Solution:

a. In 2005, $t = 15$. $P = 80(2.5)^{0.1(15)} \approx 316$ thousand. The actual population and the model's population differ by about 15,000. So, the model gives a fairly close approximation and is a good predictor of the actual population in the year 2005.

b. $t = 100$. $P = 80(2.5)^{0.1(100)} \approx 763{,}000$ thousand or 763 million. This prediction is not very realistic. It is highly unlikely for one city to be able to have this many people.

c. In 1989, $t = -1$ and in 1980, $t = -10$. In 1989, $P = 80(2.5)^{0.1(-1)} \approx 73$ thousand. A population of approximately 73,000 in 1989 seems reasonable. In 1980, $P = 80(2.5)^{0.1(-10)} = 32$ thousand. While it is possible that this city had a population of 32,000 back in 1980, it seems too small for a city. It can be concluded that for years close to 1990, the model seems to give a more reasonable population than for years farther in the past. ■

The population model in the example made good predictions for years close to and between 1990 and 1999. However, the model seemed less accurate for years farther out into the past or future. Many exponential models are valid for only a brief period of time close to when observed data is gathered to make the model.

Practice

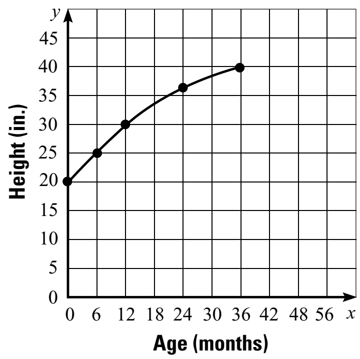
1. **Writing** An exponential model is used to predict the amount of defoliation on trees caused by a gypsy moth t years from now. Name at least two possible limitations of this model.

CHAPTER 7

Model Limitations *continued*

2. **Writing** Mrs. Wyman tracked the height of her infant daughter for 36 months, as shown in this growth chart.

Growth Chart



Would this chart be a good predictor of height at 48 months? Would it be a good predictor of height at 480 months? Explain.

3. **Writing** In 1995, the model $P = 1000(1.025)^t$ was developed to predict the student population, P, at Lexington High School t years after 1995. The population at the high school in 2005 is approximately 1800. Is the model from 1995 a good predictor of the student population in 2005? Explain.

Problem Solving

4. Beginning in 2000, new subscriptions to a magazine could be ordered through the internet. The percent of new subscriptions being processed through the internet each year has increased according to the model $S = 9(1.2)^t$, where t is the number of years since 2000. What percent of new subscriptions were processed through the internet in 2000? Use the model to predict the percent of new subscriptions that are expected in the years 2010 and 2025. What do these percents tell about the validity of this model?

5. Daniel bought a used car for $12,000 in 2002. The value of the car after it depreciates is shown in the chart below.

Year	2002	2003	2004	2005	2006
Value ($)	12,000	8500	6000	4500	2800

The value of the car can be approximated by the function $V = 12{,}000(0.7)^t$, where t is the number of years since 2002. What is the expected value of the car in 2010? If the car was new in 1997, explain whether or not this model can be used to approximate the value of the car when it was new.

6. At an accessory shop, the price of sunglasses is related to the number of sunglasses, s, the shop has available for sale each month. The price, in dollars, can be modeled according to the function $P = 100 - 0.5(2)^{0.03s}$. Last month, the shop had 100 sunglasses available for sale. Use the model to find the price of the sunglasses last month. This month, the shop has 300 sunglasses available for sale. Use the model to find the price of the sunglasses this month. What does this tell you about the validity of this model?