Name _____ Date _____

CHAPTER 7 Solving Exponential Equations

In exponential equations, if the bases are the same, then the corresponding exponents are equivalent. To solve exponential equations when the bases are the same, equate the exponents. If the bases are not the same, try rewriting them so that they are the same and then equate the exponents.

EXAMPLE 1 Rewrite numbers with a given base

Write the number using base 3.

a. 27 **b.** 81 **c.** $\frac{1}{9}$

Solution:

a. $27 = 3 \cdot 3 \cdot 3 = 3^3$ **b.** $81 = 3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ **c.** $\frac{1}{9} = \frac{1}{3 \cdot 3} = \frac{1}{3^2} = 3^{-2}$ ■

EXAMPLE 2 Solve exponential equations by inspection

Solve the equation.

a. $2^x = 8$ **b.** $9^x = 81$ **c.** $\frac{1}{25} = 5x$

Solution:

a. $8 = 2^3$ so $2^x = 2^3$ and $x = 3$.

b. $81 = 9^2$ so $9^x = 9^2$ and $x = 2$.

c. $\frac{1}{25} = 5^{-2}$ so $5^x = 5^{-2}$ and $x = -2$. ■

EXAMPLE 3 Solve exponential equations by equating the exponents

Solve the equation.

a. $4^{2x} = 64$ **b.** $5^{3x-1} = 25^x$ **c.** $2^{3-2x} = \frac{1}{16}$

Solution:

a. $64 = 4^3$, so $4^{2x} = 4^3$. Since the bases are the same, the exponents can be set equal to each other.

$2x = 3$, so $x = \frac{3}{2}$.

b. $25 = 5^2$, so $5^{3x-1} = 5^{2x}$; $3x - 1 = 2x$, so $x = 1$.

c. $\frac{1}{16} = 2^{-4}$, so $2^{3-2x} = 2^{-4}$; $3 - 2x = -4$, so $7 = 2x$ and $x = \frac{7}{2}$. ■

Pre-AP Copymasters

CHAPTER 7 Solving Exponential Equations *continued*

Sometimes both base numbers may need to be rewritten in order to solve an exponential equation.

EXAMPLE 4 Solve exponential equations by rewriting the bases

Solve the equation.

a. $9^{2x-5} = 27^{3x}$ **b.** $4^{3x+5} = 8^{4x+2}$

Solution:

a. $9 = 3^2$ and $27 = 3^3$, so $3^{2(2x-5)} = 3^{3(3x)}$;
The bases are the same, so equate the exponents.

$$2(2x - 5) = 3(3x)$$
$$4x - 10 = 9x$$
$$-10 = 5x$$
$$x = -2$$

b. $4 = 2^2$ and $8 = 2^3$, so $2^{2(3x+5)} = 2^{3(4x+2)}$.
The bases are the same, so equate the exponents.

$$2(3x + 5) = 3(4x + 2)$$
$$6x + 10 = 12x + 6$$
$$4 = 6x$$
$$x = \frac{2}{3} \blacksquare$$

Practice

Rewrite the number with a positive exponent using the smallest base possible.

1. 32 **2.** 625 **3.** 64 **4.** 243

Rewrite the number with a negative exponent using the smallest base possible.

5. $\frac{1}{36}$ **6.** $\frac{1}{1000}$ **7.** $\frac{1}{4}$ **8.** $\frac{1}{256}$

Solve the equation.

9. $2^x = 128$ **10.** $10^x = 100{,}000$ **11.** $4^x = 32$

12. $6^x = \frac{1}{216}$ **13.** $3^x = 27^{2x+1}$ **14.** $9^x = 3$

15. $16^{3x} = 4^{2x+1}$ **16.** $81^x = \frac{1}{27}$ **17.** $25^{3x-1} = 125^{x+2}$

18. $5^{2x} = 625^{3x+5}$ **19.** $\left(\frac{1}{2}\right)^{x+2} = 8$ **20.** $\left(\frac{1}{8}\right)^{2-x} = \frac{1}{16}$

21. $1000^{(2-x)} = 100^{(4-3x)}$ **22.** $\left(\frac{8}{27}\right)^{2x} = \frac{2}{3}$ **23.** $\left(\frac{125}{8}\right)^{x-3} = \left(\frac{25}{4}\right)^{2x+1}$