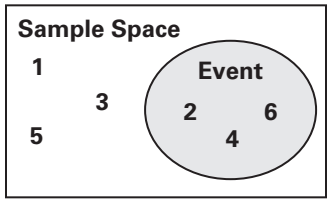
LESSON 7

Introduction to Probability

GOAL Find the probability of an event; find the odds in favor of and against an event.

A probability experiment is an activity or process involving chance. The set of all possible outcomes of a probability experiment is called the **sample space**. Recall that an event is a set consisting of one or more outcomes.

You can use Venn diagrams to represent sample spaces and events. In the Venn diagram at the right, the rectangle represents the sample space for the experiment of rolling a number cube. The event that consists of rolling an even number, {2, 4, 6}, is a subset of the sample space.



The **probability** P of an event is a measure of the likelihood that the event will occur. It is a number between 0 and 1. An event that is certain to occur has a probability of 1. An event that cannot occur has a probability of 0. An event that is equally likely to occur or not occur has a probability of $\frac{1}{2}$.

Use the formula below to find the probability of an event. In the formula, the **favorable outcomes** are the outcomes for the particular event that you are considering.

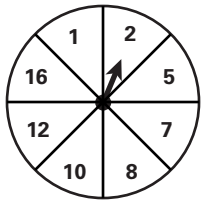
Probability of an Event

When all outcomes are equally likely, the probability P that an event will occur is given by the formula below.

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes in the sample space}}$$

EXAMPLE 1 Finding the Probability of an Event

Suppose you spin the spinner at the right, which is divided into equal parts. Find the probability of the event.



a. The pointer lands on 2.

b. The pointer lands on an even number.

SOLUTION

a. There is one favorable outcome.

$P(2) = \frac{1}{8}$ ← *There is one 2.* ← *There are 8 possible outcomes.*

The probability of landing on 2 is $\frac{1}{8}$, 0.125, or 12.5%.

b. There are 5 favorable outcomes.

$P(\text{even}) = \frac{5}{8}$ ← *There are 5 even numbers.* ← *There are 8 possible outcomes.*

The probability of landing on an even number is $\frac{5}{8}$, 0.625, or 62.5%.

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EXAMPLE 2 Probabilities Involving Permutations or Combinations

Your Spanish teacher randomly chooses 4 students to present their projects to the class. Your Spanish class has 23 students: 12 female students and 11 male students.

a. What is the probability that the 4 chosen students are all female students?

b. Suppose that Warren, Kim, Tom, and Amber are chosen. Your teacher randomly chooses an order for the 4 students to present their projects. What is the probability that they present their projects in alphabetical order by first name?

SOLUTION

a. There are ${}_{23}C_4$ different *combinations* of 4 students. Of these, there are ${}_{12}C_4$ different combinations of 4 female students.

$$P(\text{all female students}) = \frac{{}_{12}C_4}{{}_{23}C_4} = \frac{495}{8855} \approx 0.056$$

The probability that the chosen students are female is about 5.6%.

b. There are ${}_4P_4$ different *permutations* of the 4 students. Of these, there is only 1 permutation in which the students are in alphabetical order.

$$P(\text{alphabetical order}) = \frac{1}{{}_4P_4} = \frac{1}{4!} = \frac{1}{24} \approx 0.042$$

The probability that the students present their projects in alphabetical order is about 4.2%.

Types of Probability The probabilities that you found in Examples 1 and 2 are **theoretical probabilities** because they are based on knowing all of the equally likely outcomes. Probabilities based on repeated trials of an experiment are called **experimental probabilities**. An "experiment" does not have to be a scientific procedure performed in a laboratory. Experimental probabilities can also be based on surveys, historical data, or simple activities such as flipping a coin.

EXAMPLE 3 Finding Experimental Probability

To help choose a mascot for a new school, 200 students were asked to pick their favorite from the following mascots: a falcon, a lion, and a panther. The results are shown in the table at the right. Find the experimental probability that a randomly chosen student prefers the lion as the mascot.

Mascot	Votes
falcon	39
lion	76
panther	85

SOLUTION

Of the 200 students surveyed, 76 prefer the lion for the mascot.

$$P(\text{lion}) = \frac{76}{200} = 0.38$$

The probability that a randomly chosen student prefers the lion is 38%.

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CHECK **Examples 1, 2, and 3**

In Exercises 1 and 2, you randomly choose one marble from a bag that contains 6 red, 3 blue, and 7 green marbles. Find the probability of the event.

1. You choose a red marble.

2. You choose a blue marble.

In Exercises 3 and 4, each letter in ENGLISH is written on a separate piece of paper and put into a bag. You randomly choose 7 letters, one at a time, and do not replace them. Find the probability of the event.

3. The first letter chosen is a vowel.

4. The letters are chosen in alphabetical order.

5. During a season, a basketball player made 48 of the 64 free throws that she attempted. What is the probability that she will make her next free throw?

Simulations A **simulation** is an experiment that you can use to model a situation and make predictions. You can use coins, number cubes, spinners, and calculators with a random number feature to simulate events.

You should consider the *Law of Large Numbers* when conducting a simulation. The Law of Large Numbers states that as the number of trials of a simulation increases, the average of the experimental results becomes closer to the theoretical expectation. So, it is important for a simulation to have a large number of trials.

Activity

Simulating an Event

At the grand opening of a department store, each customer is given a scratch card when he or she enters the store. A discount of 10%, 15%, or 25% is revealed at the register by a store clerk when a purchase is made. Suppose that there is an equal chance of getting each discount.

1 Let the faces of a number cube represent the discounts as shown below. Roll the number cube 50 times and record your results.

Faces of the number cube	1 or 2	3 or 4	5 or 6
Discount	10%	15%	25%

2 Of the first 50 customers, how many would you expect to receive a scratch card with a discount of 10%? 15%? 25%?

3 Suppose that the discounts are not equally likely and $\frac{1}{2}$ of the cards have a 10% discount, $\frac{1}{3}$ have a 15% discount, and $\frac{1}{6}$ have a 25% discount.

How can you use a number cube to simulate the discounts given to the first 50 customers? Repeat Steps 1 and 2 for the new situation.

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Odds Another measure of the likelihood that an event will occur is **odds**. The odds in favor of an event and the odds against an event can be found using the formulas below.

$$\text{Odds in favor of an event} = \frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}}$$

$$\text{Odds against an event} = \frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}}$$

Odds are usually expressed in the form of a to b or $a : b$. For example, the odds in favor of getting a 3 when rolling a number cube are 1 to 5, or 1 : 5.

EXAMPLE 4 Finding Odds

A high school has two fall sports teams for boys and two fall sports teams for girls. The number of players on each team is listed in the table below. One student participating in fall sports is randomly chosen to represent the school at a town forum on high school sports.

Sport	football	field hockey	girls' soccer	boys' soccer
Players	39	24	18	18

a. Find the odds in favor of a field hockey player being chosen.

b. Find the odds against a football player being chosen.

c. Find the odds in favor of a soccer player being chosen.

SOLUTION

To find the odds, use the total number of students who participate in fall sports. The total is $39 + 24 + 18 + 18 = 99$.

a. There are 24 players on the field hockey team.

$$\frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}} = \frac{24}{99 - 24}$$

$$= \frac{24}{75} = \frac{8}{25}$$

The odds in favor of a field hockey player being chosen are 8 to 25.

b. There are 39 players on the football team.

$$\frac{\text{Number of unfavorable outcomes}}{\text{Number of favorable outcomes}} = \frac{99 - 39}{39}$$

$$= \frac{60}{39} = \frac{20}{13}$$

The odds against a football player being chosen are 20 to 13.

c. There are $18 + 18 = 36$ soccer players.

$$\frac{\text{Number of favorable outcomes}}{\text{Number of unfavorable outcomes}} = \frac{36}{99 - 36}$$

$$= \frac{36}{63} = \frac{4}{7}$$

The odds in favor of a soccer player being chosen are 4 to 7.

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CHECK **Example 4**

In Exercises 6 and 7, use the following information.

A juice company is running a contest in which prizes are revealed under the caps on the bottles. For every 100 bottles of juice, 30 bottles have a prize of $.50 off the next juice purchase, 5 bottles have a prize of a free bottle of juice, and 1 bottle has a prize of free admission to a movie theater. The remaining 64 bottles have no prize.

6. Find the odds in favor of winning a free bottle of juice.

7. Find the odds against winning $.50 off the next juice purchase.

EXERCISES

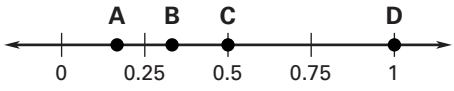
Suppose you roll a number cube. Match the event with the letter on the number line that indicates the probability of the event.

1. You roll an odd number.

2. You roll a 1.

3. You roll a number greater than 4.

4. You roll a number less than 7.



You randomly choose a tennis ball from a bucket that contains 12 yellow, 5 orange, and 3 green tennis balls. Find the probability of the event.

5. You choose a yellow tennis ball.

6. You choose an orange tennis ball.

7. You choose a green tennis ball.

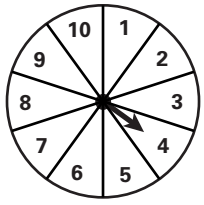
Suppose you spin the spinner below, which is divided into equal parts. Find the probability of the event.

8. The pointer lands on 3.

9. The pointer lands on a number less than 7.

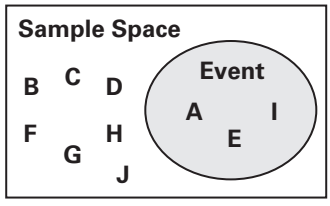
10. The pointer lands on an odd number.

11. The pointer does not land on an odd number.



In Exercises 12–14, use the following information.

A letter is written on each of 10 slips of paper. You choose a slip of paper at random. The Venn diagram shows the sample space and an event.



12. Use set notation to describe the sample space and the event.

13. Find the probability of the event.

14. Find the probability that the event does not occur.

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Decide whether you need to use *permutations* or *combinations* to find the probability. Then find the probability of the event.

15. There are 8 one-dollar bills and 4 five-dollar bills in your wallet. You randomly choose 3 bills from your wallet. What is the probability that the 3 bills you chose are all five-dollar bills?

16. Paul, James, Terry, Alex, and Ed are auditioning for the lead role in a play. The drama teacher randomly chooses an order for the 5 students to audition. What is the probability that the students audition in alphabetical order by first name?

17. You know that your uncle's telephone number has an area code of 212 and an exchange of 555, but you forgot the last four digits. If each of the four digits can be any whole number from 0 to 9, then what is the probability that your uncle's telephone number is (212)555-1234?

18. A bag contains 10 green marbles and 4 blue marbles. You randomly choose 2 marbles from the bag. What is the probability that the 2 marbles are green?

In Exercises 19 and 20, use the following information.

Students at a school were asked to name their favorite type of movie. The results are shown in the table.

Type	Students
comedy	138
drama	104
action	92
other	16

19. What is the probability that a randomly chosen student prefers comedies?

20. What is the probability that a randomly chosen student prefers dramas?

In Exercises 21–23, use the following information.

Dan rolls a number cube 60 times. The results are shown in the table below.

Outcome	1	2	3	4	5	6
Frequency	9	8	8	13	12	10

21. What is the experimental probability of rolling a number greater than 3?

22. What is the theoretical probability of rolling a number greater than 3?

23. Suppose the experimental probability for each outcome was equal to the theoretical probability. Make a table that shows these results.

In Exercises 24–26, use the following information.

You are taking a multiple choice quiz that has 10 questions. Each question has three choices, with only one of the choices being correct. You randomly choose the answers to the questions, without reading the questions.

24. Describe how you could simulate taking the quiz.

25. Conduct the simulation. How many questions did you get correct?

26. Conduct the simulation 4 more times. Then find the average number of questions answered correctly for the 5 simulations.

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In Exercises 27–29, use the following information.

A jar contains 3 black, 6 white, and 9 red marbles. You randomly choose one marble from the jar.

27. Find the odds in favor of a white marble being chosen.

28. Find the odds against a red marble being chosen.

29. Find the odds in favor of a white or a black marble being chosen.

In Exercises 30–32, use the following information.

The number of questions in each category of a trivia game are shown in the table below. You are asked the first question, which is randomly chosen by another player.

Category	movies	TV	music	literature	sports
Questions	47	43	36	34	40

30. Find the odds in favor of being asked a question about music.

31. Find the odds against being asked a question about sports.

32. Find the odds in favor of being asked a question about movies or TV.

In Exercises 33–35, use the following information.

If you know the probability that an event will occur and the probability is neither 0 nor 1, then you can find the odds by using the formulas shown below.

$$\text{Odds in favor of an event} = \frac{\text{Probability event will occur}}{1 - (\text{Probability event will occur})}$$

$$\text{Odds against an event} = \frac{1 - (\text{Probability event will occur})}{\text{Probability event will occur}}$$

33. The probability that you choose the winning number of a lottery is 0.0002. Find the odds in favor of choosing the winning number.

34. The probability that Justin will win the election for class president is 0.85. Find the odds against Justin winning the election.

35. A baseball commentator states that the probability that a National League team will win the next World Series is 0.4. Find the odds in favor of a National League team winning the next World Series.

In Exercises 36–37, use the following information.

If you know the odds in favor of or against an event, you can find the probability of the event by using the formulas shown below.

$$\text{Probability of event} = \frac{\text{Odds in favor of event}}{1 + (\text{Odds in favor of event})}$$

$$\text{Probability of event} = \frac{1}{1 + (\text{Odds against event})}$$

36. The odds in favor of choosing a yellow marble from a bag of marbles are 3 to 5. Find the probability of choosing a yellow marble.

37. The odds against rain tomorrow are 4 to 1. Find the probability that it will rain tomorrow.

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