8.1 Add and Subtract Polynomials

Before	You added and subtracted integers.
Now	You will add and subtract polynomials.
Why?	So you can model trends in recreation, as in Ex. 37.

Key Vocabulary
- monomial
- degree
- polynomial
- leading coefficient
- binomial
- trinomial

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents. The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

Monomial	Degree
10	0
$3x$	1
$\frac{1}{2}ab^2$	$1 + 2 = 3$
$-1.8m^5$	5

Not a monomial	Reason
$5 + x$	A sum is not a monomial.
$\frac{2}{n}$	A monomial cannot have a variable in the denominator.
4^a	A monomial cannot have a variable exponent.
x^{-1}	The variable must have a whole number exponent.

A **polynomial** is a monomial or a sum of monomials, each called a *term* of the polynomial. The **degree of a polynomial** is the greatest degree of its terms.

When a polynomial is written so that the exponents of a variable decrease from left to right, the coefficient of the first term is called the **leading coefficient**.

leading coefficient — degree — constant term

$$2x^3 + x^2 - 5x + 12$$

EXAMPLE 1 Rewrite a polynomial

Write $15x - x^3 + 3$ so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.

Solution

Consider the degree of each of the polynomial's terms.

Degree is 1. — Degree is 3. — Degree is 0.

$$15x - x^3 + 3$$

The polynomial can be written as $-x^3 + 15x + 3$. The greatest degree is 3, so the degree of the polynomial is 3, and the leading coefficient is -1.

BINOMIALS AND TRINOMIALS A polynomial with two terms is called a **binomial**. A polynomial with three terms is called a **trinomial**.

EXAMPLE 2 Identify and classify polynomials

Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.

	Expression	Is it a polynomial?	Classify by degree and number of terms
a.	9	Yes	0 degree monomial
b.	$2x^2 + x - 5$	Yes	2nd degree trinomial
c.	$6n^4 - 8^n$	No; variable exponent	
d.	$n^{-2} - 3$	No; negative exponent	
e.	$7bc^3 + 4b^4c$	Yes	5th degree binomial

ADDING POLYNOMIALS To add polynomials, add like terms. You can use a vertical or a horizontal format.

EXAMPLE 3 Add polynomials

Find the sum.

a. $(2x^3 - 5x^2 + x) + (2x^2 + x^3 - 1)$ **b.** $(3x^2 + x - 6) + (x^2 + 4x + 10)$

Solution

ALIGN TERMS
If a particular power of the variable appears in one polynomial but not the other, leave a space in that column, or write the term with a coefficient of 0.

a. Vertical format: Align like terms in vertical columns.

$$\begin{array}{rrrr} 2x^3 & -\ 5x^2 & +\ x & \\ +\quad x^3 & +\ 2x^2 & & -\ 1 \\ \hline 3x^3 & -\ 3x^2 & +\ x & -\ 1 \end{array}$$

b. Horizontal format: Group like terms and simplify.

$$(3x^2 + x - 6) + (x^2 + 4x + 10) = (3x^2 + x^2) + (x + 4x) + (-6 + 10)$$
$$= 4x^2 + 5x + 4$$

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✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. Write $5y - 2y^2 + 9$ so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.

2. Tell whether $y^3 - 4y + 3$ is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.

3. Find the sum $(5x^3 + 4x - 2x) + (4x^2 + 3x^3 - 6)$.

SUBTRACTING POLYNOMIALS To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by -1.

EXAMPLE 4 Subtract polynomials

Find the difference.

a. $(4n^2 + 5) - (-2n^2 + 2n - 4)$ **b.** $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

Solution

AVOID ERRORS
Remember to multiply *each* term in the polynomial by -1 when you write the subtraction as addition.

a.

$$\begin{array}{r} (4n^2 \qquad\quad + 5) \\ -(-2n^2 + 2n - 4) \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 4n^2 \qquad\quad + 5 \\ +\ 2n^2 - 2n + 4 \\ \hline 6n^2 - 2n + 9 \end{array}$$

b. $(4x^2 - 3x + 5) - (3x^2 - x - 8) = 4x^2 - 3x + 5 - 3x^2 + x + 8$

$= (4x^2 - 3x^2) + (-3x + x) + (5 + 8)$

$= x^2 - 2x + 13$

EXAMPLE 5 Solve a multi-step problem

BASEBALL ATTENDANCE Major League Baseball teams are divided into two leagues. During the period 1995–2001, the attendance N and A (in thousands) at National and American League baseball games, respectively, can be modeled by

$N = -488t^2 + 5430t + 24{,}700$ and

$A = -318t^2 + 3040t + 25{,}600$

where t is the number of years since 1995. About how many people attended Major League Baseball games in 2001?

Solution

STEP 1 **Add** the models for the attendance in each league to find a model for M, the total attendance (in thousands).

$M = (-488t^2 + 5430t + 24{,}700) + (-318t^2 + 3040t + 25{,}600)$

$= (-488t^2 - 318t^2) + (5430t + 3040t) + (24{,}700 + 25{,}600)$

$= -806t^2 + 8470t + 50{,}300$

AVOID ERRORS
Because a value of M represents *thousands* of people, $M \approx 72{,}100$ represents 72,100,000 people.

STEP 2 **Substitute** 6 for t in the model, because 2001 is 6 years after 1995.

$M = -806(6)^2 + 8470(6) + 50{,}300 \approx 72{,}100$

▶ About 72,100,000 people attended Major League Baseball games in 2001.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Find the difference $(4x^2 - 7x) - (5x^2 + 4x - 9)$.

5. **BASEBALL ATTENDANCE** Look back at Example 5. Find the difference in attendance at National and American League baseball games in 2001.

8.1 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 21 and 39

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 9, 10, 39, and 41

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A number, a variable, or the product of one or more variables is called a(n) __?__.

2. ★ **WRITING** Is 6 a polynomial? *Explain* why or why not.

EXAMPLE 1 for Exs. 3–9

REWRITING POLYNOMIALS **Write the polynomial so that the exponents decrease from left to right. Identify the degree and leading coefficient of the polynomial.**

3. $9m^5$
4. $2 - 6y$
5. $2x^2y^2 - 8xy$
6. $5n^3 + 2n - 7$
7. $5z + 2z^3 - z^2 + 3z^4$
8. $-2h^2 + 2h^4 - h^6$

9. ★ **MULTIPLE CHOICE** What is the degree of $-4x^3 + 6x^4 - 1$?

   (A) -4 (B) 3 (C) 4 (D) 6

EXAMPLE 2 for Exs. 10–16

10. ★ **MULTIPLE CHOICE** Which expression is *not* a monomial?

    (A) $-5x^2$ (B) $0.2y^4$ (C) $3mn$ (D) $3s^{-2}$

IDENTIFYING AND CLASSIFYING POLYNOMIALS **Tell whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise, tell why it is not a polynomial.**

11. -4^x
12. $w^{-3} + 1$
13. $3x - 5$
14. $\frac{4}{5}f^2 - \frac{1}{2}f + \frac{2}{3}$
15. $6 - n^2 + 5n^3$
16. $10y^4 - 3y^2 + 11$

EXAMPLES 3 and 4 for Exs. 17–28

ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

17. $(5a^2 - 3) + (8a^2 - 1)$
18. $(h^2 + 4h - 4) + (5h^2 - 8h + 2)$
19. $(4m^2 - m + 2) + (-3m^2 + 10m + 7)$
20. $(7k^2 + 2k - 6) + (3k^2 - 11k - 8)$
21. $(6c^2 + 3c + 9) - (3c - 5)$
22. $(3x^2 - 8) - (4x^3 + x^2 - 15x + 1)$
23. $(-n^2 + 2n) - (2n^3 - n^2 + n + 12)$
24. $(9b^3 - 13b^2 + b) - (-13b^2 - 5b + 14)$
25. $(4d - 6d^3 + 3d^2) - (9d^3 + 7d - 2)$
26. $(9p^2 - 6p^3 + 3 - 11p) + (7p^3 - 3p^2 + 4)$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the sum or difference of the polynomials.**

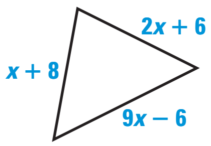
27.
$$\begin{array}{r} x^3 - 4x^2 + 3 \\ +\ \ -3x^3 + 8x - 2 \\ \hline -2x^3 + 4x^2 + 1 \end{array}$$

28.
$$\begin{aligned} &(6x^2 - 5x) - (2x^2 + 3x - 2) \\ &= (6x^2 - 2x^2) + (-5x + 3x) - 2 \\ &= 4x^2 - 2x - 2 \end{aligned}$$

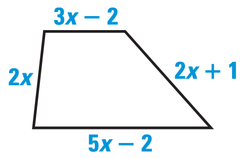
29. **ORDERING POLYNOMIALS BY DEGREE** Write the polynomials in decreasing order of *degree*: $1 - 3x + 5x^2$, $3x^5$, $x + 2x^3 + x^2$, $12x + 1$.

GEOMETRY **Write a polynomial that represents the perimeter of the figure.**

30.



31.



ADDING AND SUBTRACTING POLYNOMIALS **Find the sum or difference.**

32. $(3r^2s + 5rs + 3) + (-8rs^2 - 9rs - 12)$

33. $(x^2 + 11xy - 3y^2) + (-2x^2 - xy + 4y^2)$

34. $(5mn + 3m - 9n) - (13mn + 2m)$

35. $(8a^2b - 6a) - (2a^2b - 4b + 19)$

36. **CHALLENGE** Consider any integer x. The next consecutive integer can be represented by the binomial $(x + 1)$.

a. Write a polynomial for the sum of any two consecutive integers.

b. *Explain* how you can be sure that the sum of two consecutive integers is always odd. Use the polynomial from part (a) in your explanation.

PROBLEM SOLVING

EXAMPLE 5
for Exs. 37–39

37. **BACKPACKING AND CAMPING** During the period 1992–2002, the participation B (in millions of people) in backpacking and the participation C (in millions of people) in camping can be modeled by

$$B = -0.0262t^3 + 0.376t^2 - 0.574t + 9.67 \text{ and}$$
$$C = -0.0182t^3 + 0.522t^2 - 2.59t + 47$$

where t is the number of years since 1992. About how many more people camped than backpacked in 2002?

38. **CAR COSTS** During the period 1990–2002, the average costs D (in dollars) for a new domestic car and the average costs I (in dollars) for a new imported car can be modeled by

$$D = 442.14t + 14{,}433 \quad \text{and} \quad I = -137.63t^2 + 2705.2t + 15{,}111$$

where t is the number of years since 1990. Find the difference in average costs (in dollars) for a new imported car and a new domestic car in 2002.

39. ★ **SHORT RESPONSE** During the period 1998–2002, the number A (in millions) of books for adults and the number J (in millions) of books for juveniles sold can be modeled by

$$A = 9.5t^3 - 58t^2 + 66t + 500 \quad \text{and} \quad J = -15t^2 + 64t + 360$$

where t is the number of years since 1998.

a. Write an equation that gives the total number (in millions) of books for adults and for juveniles sold as a function of the number of years since 1998.

b. Were more books sold in 1998 or in 2002? *Explain* your answer.

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

40. SCHOOL ENROLLMENT During the period 1985–2012, the projected enrollment B (in thousands of students) in public schools and the projected enrollment R (in thousands of students) in private schools can be modeled by

$$B = -18.53t^2 + 975.8t + 48{,}140 \quad \text{and} \quad R = 80.8t + 8049$$

where t is the number of years since 1985. Write an equation that models the total school enrollment (in thousands of students) as a function of the number of years since 1985. What percent of all students is expected to be enrolled in public schools in 2012?

41. ★ EXTENDED RESPONSE The award for the best pitchers in baseball is named after the pitcher Cy Young. During the period 1890–1911, the total number of Cy Young's wins W and losses L can be modeled by

$$W = -0.44t^2 + 34t + 4.7 \quad \text{and} \quad L = 15t + 15$$

where t is the number of years since 1890.

a. A game credited to a pitcher as a win or a loss is called a decision. Write an equation that models the number of decisions for Cy Young as a function of the number of years since 1890.

b. Cy Young's career in Major League Baseball lasted from 1890 to 1911. Approximately how many total decisions did Cy Young have during his career?

c. About what percent of the decisions in Cy Young's career were wins? *Explain* how you found your answer.

Cy Young Award

42. CHALLENGE In 1970 the United States produced 63.5 quadrillion BTU (British Thermal Units) of energy and consumed 67.86 quadrillion BTU. From 1970 through 2001, the total U.S. energy production increased by about 0.2813 quadrillion BTU per year, and the total U.S. energy consumption increased by about 0.912 quadrillion BTU per year.

a. Write two equations that model the total U.S. energy production and consumption (in quadrillion BTU) as functions of the number of years since 1970.

b. How much more energy was consumed than produced in the U.S. in 1970 and in 2001? What was the change in the amount of energy consumed from 1970 to 2001?