8.2 Multiply Polynomials

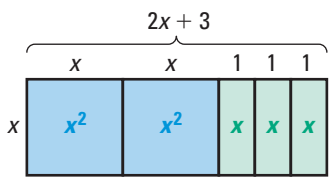
Before You added and subtracted polynomials.
Now You will multiply polynomials.
Why? So you can determine areas, as in Example 7.

Key Vocabulary
- **polynomial**
- **binomial**

The diagram shows that a rectangle with width x and length $2x + 3$ has an area of $2x^2 + 3x$. You can also find this product by using the distributive property.

$$x(2x + 3) = x(2x) + x(3) = 2x^2 + 3x$$

In this lesson, you will learn several methods for multiplying polynomials. Each method is based on the distributive property.



EXAMPLE 1 Multiply a monomial and a polynomial

Find the product $2x^3(x^3 + 3x^2 - 2x + 5)$.

$2x^3(x^3 + 3x^2 - 2x + 5)$ — Write product.

$= 2x^3(x^3) + 2x^3(3x^2) - 2x^3(2x) + 2x^3(5)$ — Distributive property

$= 2x^6 + 6x^5 - 4x^4 + 10x^3$ — Product of powers property

REVIEW PROPERTIES OF EXPONENTS
You may want to review using the properties of exponents before multiplying polynomials.

EXAMPLE 2 Multiply polynomials using a table

Find the product $(x - 4)(3x + 2)$.

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Solution

STEP 1 **Write** subtraction as addition in each polynomial.

$(x - 4)(3x + 2) = [x + (-4)](3x + 2)$

STEP 2 **Make** a table of products.

	$3x$	2
x	$3x^2$	
-4		

	$3x$	2
x	$3x^2$	$2x$
-4	$-12x$	-8

▶ The product is $3x^2 + 2x - 12x - 8$, or $3x^2 - 10x - 8$.

GUIDED PRACTICE for Examples 1 and 2

Find the product.

1. $x(7x^2 + 4)$ **2.** $(a + 3)(2a + 1)$ **3.** $(4n - 1)(n + 5)$

EXAMPLE 3 Multiply polynomials vertically

Find the product $(b^2 + 6b - 7)(3b - 4)$.

Solution

AVOID ERRORS Remember that the terms of $(3b - 4)$ are $3b$ and -4. They are *not* $3b$ and 4.

STEP 1 **Multiply** by -4.

$$\begin{array}{rr} & b^2 + 6b - 7 \\ \times & 3b - 4 \\ \hline & -4b^2 - 24b + 28 \end{array}$$

STEP 2 **Multiply** by $3b$.

$$\begin{array}{rr} & b^2 + 6b - 7 \\ \times & 3b - 4 \\ \hline & -4b^2 - 24b + 28 \\ 3b^3 + 18b^2 - 21b & \end{array}$$

STEP 3 **Add** products.

$$\begin{array}{rr} & b^2 + 6b - 7 \\ \times & 3b - 4 \\ \hline & -4b^2 - 24b + 28 \\ 3b^3 + 18b^2 - 21b & \\ \hline 3b^3 + 14b^2 - 45b + 28 & \end{array}$$

EXAMPLE 4 Multiply polynomials horizontally

Find the product $(2x^2 + 5x - 1)(4x - 3)$.

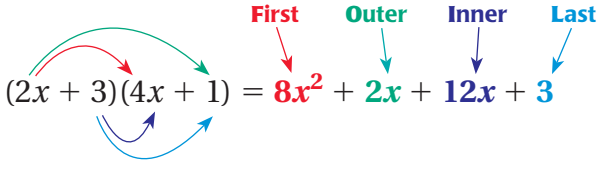
$(2x^2 + 5x - 1)(4x - 3)$ — Write product.

$= 2x^2(4x - 3) + 5x(4x - 3) - 1(4x - 3)$ — Distributive property

$= 8x^3 - 6x^2 + 20x^2 - 15x - 4x + 3$ — Distributive property

$= 8x^3 + 14x^2 - 19x + 3$ — Combine like terms.

FOIL PATTERN The letters of the word FOIL can help you to remember how to use the distributive property to multiply binomials. The letters should remind you of the words **F**irst, **O**uter, **I**nner, and **L**ast.



EXAMPLE 5 Multiply binomials using the FOIL pattern

Find the product $(3a + 4)(a - 2)$.

$(3a + 4)(a - 2)$

$= (3a)(a) + (3a)(-2) + (4)(a) + (4)(-2)$ — Write products of terms.

$= 3a^2 + (-6a) + 4a + (-8)$ — Multiply.

$= 3a^2 - 2a - 8$ — Combine like terms.

GUIDED PRACTICE for Examples 3, 4, and 5

Find the product.

4. $(x^2 + 2x + 1)(x + 2)$ **5.** $(3y^2 - y + 5)(2y - 3)$ **6.** $(4b - 5)(b - 2)$

EXAMPLE 6 Standardized Test Practice

The dimensions of a rectangle are $x + 3$ and $x + 2$. Which expression represents the area of the rectangle?

Ⓐ $x^2 + 6$ Ⓑ $x^2 + 5x + 6$ Ⓒ $x^2 + 6x + 6$ Ⓓ $x^2 + 6x$

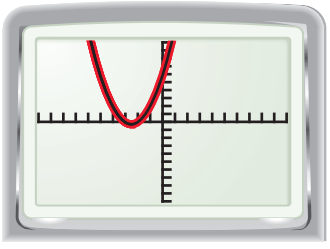
ELIMINATE CHOICES
When you multiply $x + 3$ and $x + 2$, the product will have a constant term of $3 \cdot 2 = 6$. So, you can eliminate choice D.

Solution

Area = length • width	Formula for area of a rectangle
$= (x + 3)(x + 2)$	Substitute for length and width.
$= x^2 + 2x + 3x + 6$	Multiply binomials.
$= x^2 + 5x + 6$	Combine like terms.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

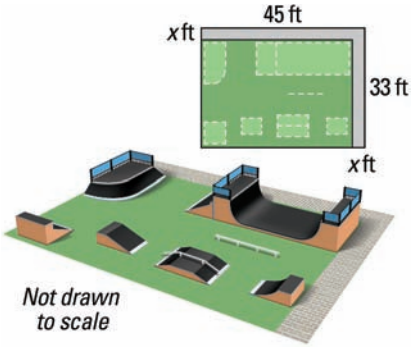
CHECK You can use a graph to check your answer. Use a graphing calculator to display the graphs of $y_1 = (x + 3)(x + 2)$ and $y_2 = x^2 + 5x + 6$ in the same viewing window. Because the graphs coincide, you know that the product of $x + 3$ and $x + 2$ is $x^2 + 5x + 6$.



EXAMPLE 7 Solve a multi-step problem

SKATEBOARDING You are designing a rectangular skateboard park on a lot that is on the corner of a city block. The park will have a walkway along two sides. The dimensions of the lot and the walkway are shown in the diagram.

- Write a polynomial that represents the area of the skateboard park.
- What is the area of the park if the walkway is 3 feet wide?



Solution

STEP 1 **Write** a polynomial using the formula for the area of a rectangle. The length is $45 - x$. The width is $33 - x$.

Area = length • width	Formula for area of a rectangle
$= (45 - x)(33 - x)$	Substitute for length and width.
$= 1485 - 45x - 33x + x^2$	Multiply binomials.
$= 1485 - 78x + x^2$	Combine like terms.

STEP 2 **Substitute** 3 for x and evaluate.

$$\text{Area} = 1485 - 78(3) + (3)^2 = 1260$$

▶ The area of the park is 1260 square feet.

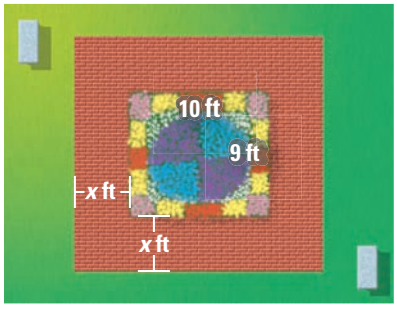
✓ GUIDED PRACTICE for Examples 6 and 7

7. The dimensions of a rectangle are $x + 5$ and $x + 9$. Which expression represents the area of the rectangle?

Ⓐ $x^2 + 45x$ Ⓑ $x^2 + 45$

Ⓒ $x^2 + 14x + 45$ Ⓓ $x^2 + 45x + 45$

8. GARDEN DESIGN You are planning to build a walkway that surrounds a rectangular garden, as shown. The width of the walkway around the garden is the same on every side.



a. Write a polynomial that represents the combined area of the garden and the walkway.

b. Find the combined area when the width of the walkway is 4 feet.

8.2 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 23 and 51

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 26, 44, 52, and 53

SKILL PRACTICE

1. VOCABULARY Copy and complete: The FOIL pattern can be used to multiply any two ___?___.

2. ★ WRITING *Explain* how the letters of the word FOIL can help you multiply polynomials.

EXAMPLE 1 for Exs. 3–8

MULTIPLYING POLYNOMIALS **Find the product.**

3. $x(2x^2 - 3x + 9)$ **4.** $4y(-y^3 - 2y - 1)$ **5.** $z^2(4z^4 + z^3 - 11z^2 - 6)$

6. $3c^3(8c^4 - c^2 - 3c + 5)$ **7.** $-a^5(-9a^2 + 5a + 13)$ **8.** $-5b^3(4b^5 - 2b^3 + b - 11)$

EXAMPLE 2 for Exs. 9–15

USING TABLES **Use a table to find the product.**

9. $(x + 2)(x - 3)$ **10.** $(y - 5)(2y + 3)$ **11.** $(4b - 3)(b - 7)$

12. $(5s + 2)(s + 8)$ **13.** $(3k - 1)(4k + 9)$ **14.** $(8n - 5)(3n - 6)$

EXAMPLES 3 and 4 for Exs. 16–26

ERROR ANALYSIS ***Describe*** **and correct the error in finding the product of the polynomials.**

15.

$(x - 5)(3x + 1)$

	$3x$	1
x	$3x^2$	x
5	$15x$	5

$(x - 5)(3x + 1) = 3x^2 + 16x + 5$

16.

$$\begin{array}{r} 2x^2 - 3x - 4 \\ \times \quad\quad x + 7 \\ \hline 14x^2 - 21x - 28 \\ 2x^3 - 3x^2 - 4x \\ \hline 2x^3 + 11x^4 - 25x^2 - 28 \end{array}$$

MULTIPLYING POLYNOMIALS **Use a vertical or a horizontal format to find the product.**

17. $(y + 6)(y - 5)$ **18.** $(5x - 8)(2x - 5)$ **19.** $(7w + 5)(11w - 3)$

20. $(b - 2)(b^2 - b + 1)$ **21.** $(s + 4)(s^2 + 6s - 5)$ **22.** $(-r + 7)(2r^2 - r - 9)$

23. $(5x + 2)(-3x^2 + 4x - 1)$ **24.** $(y^2 + 8y - 6)(4y - 3)$ **25.** $(6z^2 + z - 1)(9z - 5)$

26. ★ **MULTIPLE CHOICE** What is the product of $2x - 9$ and $4x + 1$?

Ⓐ $8x^2 - 38x - 9$ Ⓑ $8x^2 - 34x - 9$

Ⓒ $8x^2 + 34x - 9$ Ⓓ $8x^2 + 38x - 9$

EXAMPLE 5 for Exs. 27–32

USING THE FOIL PATTERN **Use the FOIL pattern to find the product.**

27. $(2r - 1)(5r + 3)$ **28.** $(7a - 2)(3a - 4)$ **29.** $(4m + 9)(2m + 7)$

30. $(8t + 11)(6t - 1)$ **31.** $(4x - 5)(12x - 7)$ **32.** $(8z + 3)(5z + 4)$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

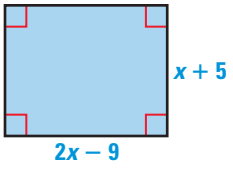
33. $p(2p - 3) + (p - 3)(p + 3)$ **34.** $x^2(7x + 5) - (2x + 6)(x - 1)$

35. $-3c^2(c + 11) - (4c - 5)(3c - 2)$ **36.** $2w^3(2w^3 - 7w - 1) + w(5w^2 + 2w)$

EXAMPLES 6 and 7 for Exs. 37–42

GEOMETRY **Write a polynomial that represents the area of the shaded region.**

37.



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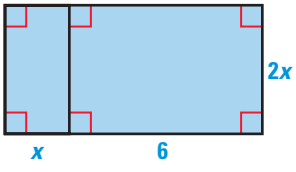

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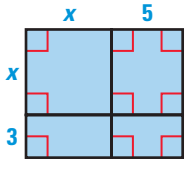

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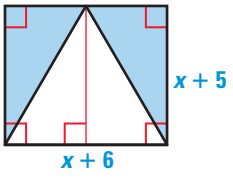

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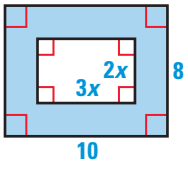
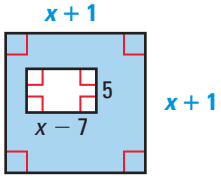


42.



43. **POLYNOMIAL FUNCTIONS** Find the product $f(x) \cdot g(x)$ for the functions $f(x) = x - 11$ and $g(x) = 2x + 12$.

44. ★ **MULTIPLE CHOICE** Which polynomial represents $f(x) \cdot g(x)$ if $f(x) = -2x^2$ and $g(x) = x^3 - 5x^2 + 2x - 1$?

Ⓐ $-2x^5 - 10x^4 + 4x^3 - 2x^2$ Ⓑ $-2x^5 + 10x^4 - 4x^3 - 2x^2$

Ⓒ $-2x^5 + 10x^4 - 4x^3 + 2x^2$ Ⓓ $2x^5 - 10x^4 + 4x^3 - 2x^2$

45. **REASONING** Find the product $(x^2 - 7x)(2x^2 + 3x + 1)$. Show that the product is correct by using a graphing calculator. *Explain* your reasoning.

CHALLENGE **Find the product.**

46. $(x - y)(3x + 4y)$ **47.** $(x^2y + 9y)(2x + 3y)$ **48.** $(x^2 - 5xy + y^2)(4xy)$

◯ = See WORKED-OUT SOLUTIONS in Student Resources

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

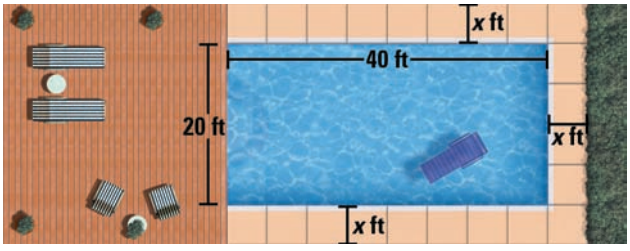
EXAMPLE 7 for Exs. 49–50

49. PICTURE FRAME You are designing a frame to surround a rectangular picture. The width of the frame around the picture is the same on every side, as shown.

a. Write a polynomial that represents the total area of the picture and the frame.

b. Find the combined area of the picture and the frame when the width of the frame is 4 inches.

50. SWIMMING POOL A rectangular swimming pool is bordered on one side by a deck. A contractor is hired to build a walkway along the remaining three sides of the pool. The width of the walkway is the same on every side, as shown.



a. Write a polynomial that represents the total area of the pool and the walkway.

b. Find the combined area of the pool and the walkway when the width of the walkway is 5 feet.

51. SOUND RECORDINGS During the period 1997–2002, the amount of money R (in millions of dollars) spent on sound recordings in the U.S. and the percent P (in decimal form) of this amount spent by people who are between 15 and 19 years old can be modeled by

$$R = -336t^2 + 1730t + 12{,}300 \text{ and } P = 0.00351t^2 - 0.0249t + 0.171$$

where t is the number of years since 1997.

a. Find the values of R and P for $t = 0$. What does the product $R \cdot P$ mean for $t = 0$ in this situation?

b. Write an equation that models the amount spent on sound recordings by people who are between 15 and 19 years old as a function of the number of years since 1997.

c. How much money did people between 15 and 19 years old spend on sound recordings in 2002?

52. ★ SHORT RESPONSE During the period 1980–2002, the number H (in thousands) of housing units in the U.S. and the percent P (in decimal form) of housing units that were vacant can be modeled by

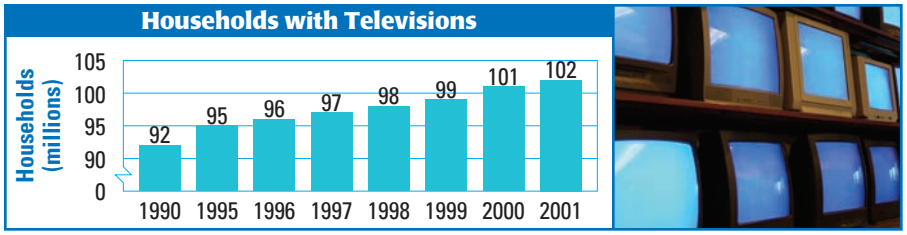
$$H = 1570t + 89{,}000 \quad \text{and} \quad P = 0.0013t + 0.094$$

where t is the number of years since 1980.

a. Write an equation that models the number (in thousands) of vacant housing units as a function of the number of years since 1980. *Explain* how you found this equation.

b. How many housing units were vacant in 2002?

53. ★ **EXTENDED RESPONSE** The bar graph shows the number of households with a television for various years during the period 1990–2001.



a. Find a linear equation that models the number of households T (in millions) with a television as a function of the number of years since 1990. *Explain* how you found your model.

b. During the period 1990–2001, the percent P (in decimal form) of television households that also have a VCR can be modeled by

$$P = -0.0015t^2 + 0.032t + 0.069$$

where t is the number of years since 1990. Write an equation that models the number of households V (in millions) with a VCR and a television as a function of the number of years since 1990.

c. Use the equation from part (b) to predict the number of households that had a VCR and a television in 2002 and in 2005.

54. **CHALLENGE** For the period 1990–2001, the total United States energy consumption C (in quadrillion British Thermal Units, or BTU) and the percent P of the total energy that was consumed in the United States for industrial purposes can be modeled by

$$C = 1.5t + 84$$

$$P = -0.05t^2 + 0.25t + 38$$

where t is the number of years since 1990.

a. Find the percent of total energy that was consumed in the United States for industrial purposes in 2000.

b. Write an equation that gives the total energy (in quadrillion BTU) consumed in the United States for industrial purposes as a function of the number of years since 1990. To write the equation, you may need to rewrite one of the given equations.