Name ______________________ Date ____________

CHAPTER 8

A Multiplication Strategy

You learned to multiply polynomials horizontally, vertically, using a table, and using the FOIL pattern. With each method, each term of the first polynomial is multiplied by each term of the second polynomial. As the number of terms in each polynomial increases, the need for organizing this multiplication becomes more important. You can use the methods learned for multiplying polynomials as tools to help with this organization.

EXAMPLE 1 Organize polynomial products

Find the product $(2x - 1)(3x^3 + 4x^2 - 5x - 2)$ horizontally and using a table.

Solution:

Horizontally:

$$\begin{aligned}(2x - 1)(3x^3 + 4x^2 - 5x - 2) &= 2x(3x^3 + 4x^2 - 5x - 2) - 1(3x^3 + 4x^2 - 5x - 2)\\ &= 2x(3x^3) + 2x(4x^2) + 2x(-5x) + 2x(-2) - 1(3x^3) - 1(4x^2) - 1(-5x) - 1(-2)\\ &= 6x^4 + 8x^3 - 10x^2 - 4x - 3x^3 - 4x^2 + 5x + 2\\ &= 6x^4 + 5x^3 - 14x^2 + x + 2\end{aligned}$$

Using a table:

	$3x^3$	$4x^2$	$-5x$	-2
$2x$	$6x^4$	$8x^3$	$-10x^2$	$-4x$
-1	$-3x^3$	$-4x^2$	$5x$	2

$6x^4 + 8x^3 - 10x^2 - 4x - 3x^3 - 4x^2 + 5x + 2 = 6x^4 + 5x^3 - 14x^2 + x + 2$ ■

Regardless of which method is chosen, the same product results. Organization is key in helping to achieve the correct product.

In both methods used in Example 1, the two terms in the first polynomial were multiplied by the four terms in the second polynomial. The total number of multiplications that took place was 2×4 or 8. This can be verified by counting the multiplications in the horizontal method or seen visually by the 2×4 table in which the 8 spaces are filled.

When multiplying binomials by binomials, the FOIL method provides a convenient method for keeping the multiplication well organized.

KEY CONCEPT

Multiplications of Polynomials

Multiplying a polynomial with m terms by a polynomial with n terms will involve mn total multiplications.

EXAMPLE 2 Find the number of multiplications

Find the total number of multiplications involved.

a. $(n^2 + 3)(4n + 1)$

b. $(2d^5 - 6d^4 + d^2 + 4)(d^2 - 5d - 5)$

c. $6k^2(3k^4 + 5k^2 - 2k^2 - 7k - 8)$

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CHAPTER 8

A Multiplication Strategy *continued*

Solution:

a. 2 terms in the first polynomial × 2 terms in the second polynomial = 4 total multiplications

b. 4 terms in the first polynomial × 3 terms in the second polynomial = 12 total multiplications

c. 1 term in the first polynomial × 5 terms in the second polynomial = 5 total multiplications ■

The number of terms in a polynomial product is not the same as the number of multiplications.

KEY CONCEPT

Multiplications of Polynomials

The number of terms in a polynomial product with mn multiplications will be less than or equal to mn. This is because in many polynomial products, like terms are combined. Only when there are no like terms to combine will there be a total of mn terms in the product.

Practice

Find the number of multiplications involved.

1. $w^2(3w^2 + 5w + 1)$

2. $(2q^5 - 7q^4)(2q^2 - 15q + 8)$

3. $(a^4 + 3a^3 + 6a^2 - 4)(3a^2 - 5a)$

4. $(2d^2 + 4g^2 - g)(d^2 + d - g)$

Find the number of terms in the simplified product.

5. $(2k^5 - 3k^3 + k)(k^2 - 4)$

6. $(r^3 + r^2 + 2r + 2)(r^2 - 2r - 2)$

7. $(x^2 - 2y^2)(x + 3y^2)$

8. $(2s^4 - 6s^2 + t^2)(s^2 + 6t + 1)$

Find the product.

9. $(u^4 - u^3 - u^2)(2u^2 - 3u + 2)$

10. $(2b^5 - 5b^4 + b^2 + 4b)(-5b^3 - 1)$

11. $(4p^5 + p^3 + n^4 + 4n^2)(p^3 + 3n^2 + 3)$

12. $(v^2 - 7z^2 + v - 2)(7v^2 + z^2 + 2v - z)$

Problem Solving

13. Four congruent squares are cut from each corner of a 30 centimeter by 50 centimeter piece of cardboard. The sides of the cardboard are then folded up to form a box. The length of each cut square is $2x$ centimeters. What are the dimensions of the box? Find an expression for the volume of the box in cubic centimeters.

14. Two years ago, Paul's rectangular garden was $x + 3$ feet long and $2x - 1$ feet wide. Last year, he lengthened each dimension by $4y - 5$ feet. This year, he lengthened the dimensions by another $z + 6$ feet. What are the new dimensions of Paul's garden? Find an expression for the area, in square feet, of the garden this year.