Name ______________________________ Date ______________

CHAPTER 8

# More Special Products

Finding the cube of a binomial can be done in more than one way. One of the ways is to multiply the binomial by itself three times.

$$(a + b)^3 = (a + b)(a + b)(a + b)$$

Start by multiplying the first two binomials together.

$$(a + b)(a + b) = a^2 + 2ab + b^2$$

Multiply this result by the third binomial.

$$(a^2 + 2ab + b^2)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$$

Another way to find the product is to use the cube of a binomial pattern.

**KEY CONCEPT**

## Cube of a Binomial Pattern

| Algebra | Example |
|---|---|
| $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ | $(x + 4)^3 = x^3 + 12x^2 + 48x + 64$ |
| $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ | $(3x - 2)^3 = 27x^3 - 54x^2 + 36x - 8$ |

Recall that $(a + b)^2 \neq a^2 + b^2$ and $(a - b)^2 \neq a^2 - b^2$. That is because $(a + b)^2 = (a + b)(a + b)$ and $(a - b)^2 = (a - b)(a - b)$.

Notice, too, that $(a + b)^3 \neq a^3 + b^3$ and $(a - b)^3 \neq a^3 - b^3$ since $(a + b)^3 = (a + b)(a + b)(a + b)$ and $(a - b)^3 = (a - b)(a - b)(a - b)$. Each binomial is multiplied together to find the product.

**EXAMPLE 1**

## Use the cube of a binomial pattern

Find the product.

**a.** $(2x + 5)^3$

**b.** $(4x - 1)^3$

**Solution:**

**a.** $(2x + 5)^3 = (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + 5^3$
$= 8x^3 + 60x^2 + 150x + 125$

**b.** $(4x - 1)^3 = (4x)^3 - 3(4x)^2(1) + 3(4x)(1)^2 - 1^3$
$= 64x^3 + 48x^2 + 12x - 1$ ■

Trinomials can be squared in a similar way. Grouping trinomials such as $(a + b + c)^2 = [(a + b) + c]^2$ allows for the square of a binomial pattern to be used, as shown in Example 2.

**EXAMPLE 2**

## Square trinomials by grouping

Find the product.

**a.** $(x^2 + 4x + 3)^2$

**b.** $(6x - 2y - z)^2$

**Solution**

**a.** $(x^2 + 4x + 3)^2 = [(x^2 + 4x) + 3]^2$

$= (x^2 + 4x)^2 + 2(x^2 + 4x)(3) + 3^2$

$= (x^2)^2 + 2(x^2)(4x) + (4x)^2 + 6x^2 + 24x + 9$

$= x^4 + 8x^3 + 16x^2 + 6x^2 + 24x + 9$

$= x^4 + 8x^3 + 22x^2 + 24x + 9$

**b.** $(6x - 2y - z)^2 = [(6x - 2y) - z]^2$

$= (6x - 2y)^2 - 2(6x - 2y)(z) + z^2$

$= (6x)^2 - 2(6x)(2y) + (2y)^2 - 12xz + 4yz + z^2$

$= 36x^2 - 24xy + 4y^2 - 12xz + 4yz + z^2$ ■

## Practice

1. **Writing** Explain why $(x + 3)^2 \neq x^2 + 9$.
2. **Writing** Explain why $(3x - 2y)^3 \neq 9x^3 - 4y^3$.
3. **Writing** Explain whether or not the product of $(x + y + z)^2$ is $x^2 + y^2 + z^2$. Justify your reasoning.

**Find the cube of the binomial.**

4. $(m + 2)^3$
5. $(t - 7)^3$
6. $(5k + 6)^3$
7. $(2q - 1)^3$
8. $(c + d)^3$
9. $(f - g)^3$
10. $(w + 2z)^3$
11. $(4r - 3s)^3$
12. $(-7v - 2w)^3$

**Find the square of the trinomial.**

13. $(x^2 + 6x + 4)^2$
14. $(h^2 - 2h + 7)^2$
15. $(3j^2 - j + 4)^2$
16. $(x - 3y - 2)^2$
17. $(4y + 3w - 2z)^2$
18. $(5m - 3r - 2q)^2$

**Problem Solving**

19. A moving box is shaped like a cube. Each side of the moving box is $4x - 3$ inches long. Write a polynomial that gives the volume of the moving box in cubic inches.
20. A square area of the gymnasium floor has been marked off for a sports tournament. The length of each side in feet is given by $2x^2 + 5x - 3$. Write a polynomial that gives the area, in square feet, of the floor marked off for the sports tournament.