8.3 Find Special Products of Polynomials

Before You multiplied polynomials.

Now You will use special product patterns to multiply polynomials.

Why? So you can make a scientific prediction, as in Example 4.

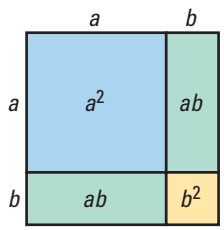
Key Vocabulary
- **binomial**
- **trinomial**

COMMON CORE

CC.9-12.A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

The diagram shows a square with a side length of $(a + b)$ units. You can see that the area of the square is

$$(a + b)^2 = a^2 + 2ab + b^2.$$



This is one version of a pattern called the square of a binomial. To find another version of this pattern, use algebra: replace b with $-b$.

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$ **Replace b with $-b$ in the pattern above.**

$(a - b)^2 = a^2 - 2ab + b^2$ **Simplify.**

KEY CONCEPT *For Your Notebook*

Square of a Binomial Pattern

Algebra	Example
$(a + b)^2 = a^2 + 2ab + b^2$	$(x + 5)^2 = x^2 + 10x + 25$
$(a - b)^2 = a^2 - 2ab + b^2$	$(2x - 3)^2 = 4x^2 - 12x + 9$

EXAMPLE 1 Use the square of a binomial pattern

USE PATTERNS
When you use special product patterns, remember that a and b can be numbers, variables, or variable expressions.

Find the product.

a. $(3x + 4)^2 = (3x)^2 + 2(3x)(4) + 4^2$ **Square of a binomial pattern**

$= 9x^2 + 24x + 16$ **Simplify.**

b. $(5x - 2y)^2 = (5x)^2 - 2(5x)(2y) + (2y)^2$ **Square of a binomial pattern**

$= 25x^2 - 20xy + 4y^2$ **Simplify.**

GUIDED PRACTICE for Example 1

Find the product.

1. $(x + 3)^2$ **2.** $(2x + 1)^2$ **3.** $(4x - y)^2$ **4.** $(3m + n)^2$

SUM AND DIFFERENCE PATTERN To find the product $(x + 2)(x - 2)$, you can multiply the two binomials using the FOIL pattern.

$$(x + 2)(x - 2) = x^2 - 2x + 2x - 4 \qquad \text{Use FOIL pattern.}$$
$$= x^2 - 4 \qquad \text{Combine like terms.}$$

This suggests a pattern for the product of the sum and difference of two terms.

KEY CONCEPT *For Your Notebook*

Sum and Difference Pattern

Algebra	Example
$(a + b)(a - b) = a^2 - b^2$	$(x + 3)(x - 3) = x^2 - 9$

EXAMPLE 2 Use the sum and difference pattern

Find the product.

a. $(t + 5)(t - 5) = t^2 - 5^2$ — Sum and difference pattern

$= t^2 - 25$ — Simplify.

b. $(3x + y)(3x - y) = (3x)^2 - y^2$ — Sum and difference pattern

$= 9x^2 - y^2$ — Simplify.

GUIDED PRACTICE for Example 2

Find the product.

5. $(x + 10)(x - 10)$ **6.** $(2x + 1)(2x - 1)$ **7.** $(x + 3y)(x - 3y)$

SPECIAL PRODUCTS AND MENTAL MATH The special product patterns can help you use mental math to find certain products of numbers.

EXAMPLE 3 Use special products and mental math

Use special products to find the product 26 • 34.

Solution

Notice that 26 is 4 less than 30 while 34 is 4 more than 30.

$26 \cdot 34 = (30 - 4)(30 + 4)$ — Write as product of difference and sum.

$= 30^2 - 4^2$ — Sum and difference pattern

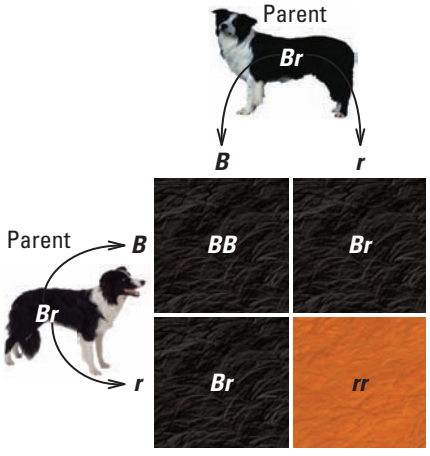
$= 900 - 16$ — Evaluate powers.

$= 884$ — Simplify.

EXAMPLE 4 Solve a multi-step problem

BORDER COLLIES The color of the dark patches of a border collie's coat is determined by a combination of two genes. An offspring inherits one patch color gene from each parent. Each parent has two color genes, and the offspring has an equal chance of inheriting either one.

The gene B is for black patches, and the gene r is for red patches. Any gene combination with a B results in black patches. Suppose each parent has the same gene combination Br. The Punnett square shows the possible gene combinations of the offspring and the resulting patch color.



- What percent of the possible gene combinations of the offspring result in black patches?
- Show how you could use a polynomial to model the possible gene combinations of the offspring.

Solution

STEP 1 **Notice** that the Punnett square shows 4 possible gene combinations of the offspring. Of these combinations, 3 result in black patches.

▶ 75% of the possible gene combinations result in black patches.

STEP 2 **Model** the gene from each parent with $0.5B + 0.5r$. There is an equal chance that the collie inherits a black or red gene from each parent.

The possible genes of the offspring can be modeled by $(0.5B + 0.5r)^2$. Notice that this product also represents the area of the Punnett square.

Expand the product to find the possible patch colors of the offspring.

$$(0.5B + 0.5r)^2 = (0.5B)^2 + 2(0.5B)(0.5r) + (0.5r)^2$$
$$= 0.25B^2 + 0.5Br + 0.25r^2$$

Consider the coefficients in the polynomial.

$$0.25B^2 + 0.5Br + 0.25r^2$$

25% *BB*, black patches | 50% *Br*, black patches | 25% *rr*, red patches

The coefficients show that 25% + 50% = 75% of the possible gene combinations will result in black patches.

GUIDED PRACTICE for Examples 3 and 4

8. *Describe* how you can use special products to find 21^2.

9. **BORDER COLLIES** Look back at Example 4. What percent of the possible gene combinations of the offspring result in red patches?

8.3 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 11 and 41
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 17, 18, 42, and 44
- = **MULTIPLE REPRESENTATIONS** Ex. 41

SKILL PRACTICE

1. **VOCABULARY** Give an example of two binomials whose product you can find using the sum and difference pattern.

2. ★ **WRITING** *Explain* how to use the square of a binomial pattern.

EXAMPLE 1 for Exs. 3–10, 18

SQUARE OF A BINOMIAL **Find the product.**

3. $(x + 8)^2$
4. $(a + 6)^2$
5. $(2y + 5)^2$
6. $(t - 7)^2$
7. $(n - 11)^2$
8. $(6b - 1)^2$

ERROR ANALYSIS ***Describe*** **and correct the error in multiplying.**

9. $(s - 3)^2 = s^2 + 9$ ✗

10. $(2d - 10)^2 = 4d^2 - 20d + 100$ ✗

EXAMPLE 2 for Exs. 11–17

SUM AND DIFFERENCE PATTERN **Find the product.**

11. $(t + 4)(t - 4)$
12. $(m - 6)(m + 6)$
13. $(2x + 1)(2x - 1)$
14. $(3x - 1)(3x + 1)$
15. $(7 + w)(7 - w)$
16. $(3s - 8)(3s + 8)$

17. ★ **MULTIPLE CHOICE** Find the product $(7x + 3)(7x - 3)$.

Ⓐ $7x^2 - 9$ Ⓑ $49x^2 - 9$ Ⓒ $49x^2 - 21x - 9$ Ⓓ $49x^2 - 42x - 9$

18. ★ **MULTIPLE CHOICE** Find the product $(5n - 3)^2$.

Ⓐ $5n^2 - 9$ Ⓑ $25n^2 - 9$ Ⓒ $25n^2 - 15n + 9$ Ⓓ $25n^2 - 30n + 9$

EXAMPLE 3 for Exs. 19–22

MENTAL MATH ***Describe*** **how you can use mental math to find the product.**

19. $16 \cdot 24$
20. $28 \cdot 32$
21. 17^2
22. 44^2

SPECIAL PRODUCT PATTERNS **Find the product.**

23. $(r + 9s)^2$
24. $(6x + 5)^2$
25. $(3m + 11n)(3m - 11n)$
26. $(7a + 8b)(7a - 8b)$
27. $(3m - 7n)^2$
28. $(13 - 2x)^2$
29. $(3f - 9)(3f + 9)$
30. $(9 - 4t)(9 + 4t)$
31. $(3x + 8y)^2$
32. $(-x - 2y)^2$
33. $(2a - 5b)(2a + 5b)$
34. $(6x + y)(6x - y)$

MULTIPLYING FUNCTIONS **Perform the indicated operation using the functions $f(x) = 3x + 0.5$ and $g(x) = 3x - 0.5$.**

35. $f(x) \cdot g(x)$
36. $(f(x))^2$
37. $(g(x))^2$

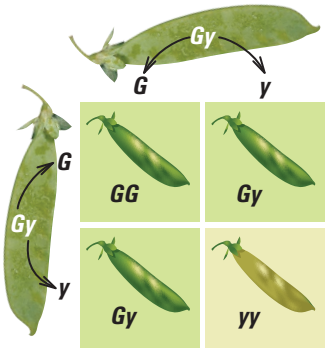
38. **CHALLENGE** Write two binomials that have the product $x^2 - 121$. *Explain*.

39. **CHALLENGE** Write a pattern for the cube of a binomial $(a + b)^3$.

PROBLEM SOLVING

EXAMPLE 4 for Exs. 40–42

40. PEA PLANTS In pea plants, the gene G is for green pods, and the gene y is for yellow pods. Any gene combination with a G results in a green pod. Suppose two pea plants have the same gene combination Gy. The Punnett square shows the possible gene combinations of an offspring pea plant and the resulting pod color.



a. What percent of possible gene combinations of the offspring plant result in a yellow pod?

b. Show how you could use a polynomial to model the possible gene combinations of the offspring.

41. MULTIPLE REPRESENTATIONS In humans, the gene s is for straight thumbs, and the gene C is for curved thumbs. Any gene combination with a C results in a curved thumb. Suppose each parent has the same gene combination Cs.

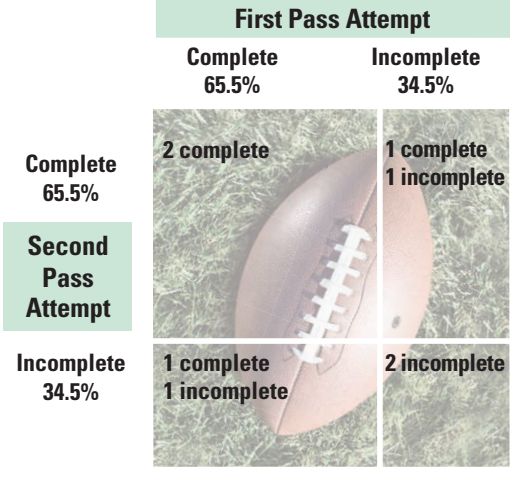
a. Making a Diagram Make a Punnett square that shows the possible gene combinations inherited by a child.

b. Writing a Model Write a polynomial that models the possible gene combinations of the child.

c. Interpreting a Model What percent of the possible gene combinations of the child result in a curved thumb?

42. ★ SHORT RESPONSE In ball pythons, the gene N is for normal coloring, and the gene a is for no coloring, or albino. Any gene combination with an N results in normal coloring. Suppose one parent python has the gene combination Na and the other parent python has the gene combination aa. What percent of the possible gene combinations of the offspring result in an albino python? *Explain* how you found your answer.

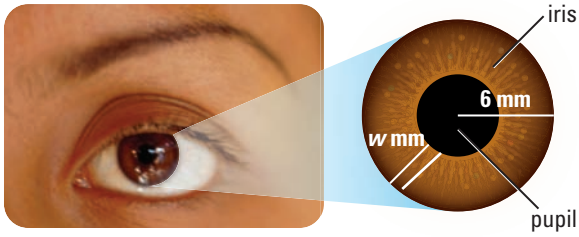
43. FOOTBALL STATISTICS During the 2004 regular season, the San Diego Chargers' quarterback Drew Brees completed 65.5% of the passes he attempted. The area model shows the possible outcomes of two attempted passes.



a. What percent of the possible outcomes of two attempted passes results in Drew Brees's throwing at least one complete pass? *Explain* how you found your answer using the area model.

b. Show how you could use a polynomial to model the possible results of two attempted passes.

44. ★ **EXTENDED RESPONSE** The iris of an eye surrounds the pupil. It regulates the amount of light entering the eye by opening and closing the pupil. For parts (a)–(c) below, leave your answers in terms of π.



The iris of a human eye has a width *w* that varies from 0.5 millimeter to 4 millimeters.

a. Write a polynomial that represents the pupil's radius.

b. Write a polynomial that represents the pupil's area.

c. What is the least possible area and the greatest possible area of the pupil? *Explain* how you found your answers.

45. **CHALLENGE** You use 100 feet of fencing to form a square with a side length of 25 feet. You want to change the dimensions of the enclosed region. For every 1 foot you increase the width, you must decrease the length by 1 foot. Write a polynomial that gives the area of the rectangle after you increase the width by x feet and decrease the length by x feet. *Explain* why *any* change in dimensions results in an area less than that of the original square.

QUIZ

Find the sum, difference, or product.

1. $(x^2 - 3x + 5) + (-2x^2 + 11x + 1)$

2. $(8y^3 - 7y^2 + y) - (9y^2 - 5y + 7)$

3. $(2r + 11)(r - 6)$

4. $(m + 3)(-2m^2 + 5m - 1)$

5. $(2 + 8p)(2 - 10p)$

6. $(15 - 2s)^2$

7. $(5w + 9z)^2$

8. $(5x - 4y)(5x + 4y)$

9. **AREA** The length of a rectangular rug is 2 times its width. The rug is centered in a rectangular room. Each edge is 3 feet from the nearest wall. Write a polynomial that represents the area of the room.