GOAL

Solve polynomial equations.

Vocabulary

The zero-product property is used to solve an equation when one side is zero and the other side is a product of polynomial factors. The solutions of such an equation are also called **roots**.

The height of a projectile can be described by the **vertical motion model:** $h = -16t^2 + vt + s$, where t is the time (in seconds) the object has been in the air, v is the initial vertical velocity (in feet per second), and s is the initial height (in feet).

EXAMPLE 1

Use the zero-product property

Solve
$$(x - 3)(x + 6) = 0$$
.

Solution

$$(x-3)(x+6) = 0$$

(x-3)(x+6) = 0 Write original equation.

$$x - 3 = 0$$
 or $x + 6 = 0$ Zero-product property

$$x = 3$$
 or $x = -6$

Solve for x.

The roots of the equation are 3 and -6.

CHECK Substitute each root into the original equation to check.

$$(3-3)(3+6) \stackrel{?}{=} 0$$
 $(-6-3)(-6+6) \stackrel{?}{=} 0$

$$-6-3)(-6+6) =$$

$$0 \cdot 9 \stackrel{?}{=} 0$$

$$-9 \cdot 0 \stackrel{?}{=} 0$$

$$0 = 0$$
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$$0 = 0$$
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Exercises for Example 1

Solve the equation.

1.
$$(m-7)(m-9)=0$$

2.
$$(5n + 10)(4n + 12) = 0$$

EXAMPLE 2 Solve an equation by factoring

Solve $6x^2 + 12x = 0$.

$$6x^2 + 12x = 0$$

Write original equation.

$$6x(x+2)=0$$

Factor left side.

$$6x = 0 \quad or \quad x + 2 = 0$$

Zero-product property

$$x = 0$$
 or $x = -2$

$$x = -2$$

The roots of the equation are 0 and -2.

Study Guide continued

For use with the lesson "Solve Polynomial Equations in Factored Form"

Solve an equation by factoring **EXAMPLE 3**

Solve $9v^2 = 21v$.

$$9y^2 = 21y$$

Write original equation.

$$9y^2 - 21y = 0$$

Subtract 21v from each side.

$$3y(3y-7)=0$$

Factor left side.

$$3y = 0 \quad or \quad 3y - 7 = 0$$

Zero-product property

$$y = 0$$
 or $y = \frac{7}{3}$ Solve for y.

$$y = \frac{7}{3}$$

The roots of the equation are 0 and $\frac{1}{3}$.

Exercises for Examples 2 and 3

Solve the equation.

3.
$$q^2 + 16q = 0$$

4.
$$4k^2 - 8k = 0$$
 5. $12h^2 = 36h$

5.
$$12h^2 = 36h$$

EXAMPLE 4 Solve a multi-step problem

Jump Rope A child jumping rope leaves the ground at an initial vertical velocity of 8 feet per second. After how many seconds does the child land on the ground?

Solution

STEP 1 Write a model for the height above the ground.

$$h = -16t^2 + vt + s$$

Vertical motion model

$$h = -16t^2 + 8t + 0$$

Substitute 8 for v and 0 for s.

$$h = -16t^2 + 8t$$

Simplify.

STEP 2 Substitute 0 for h. When the child lands, the child's height above the ground is 0 feet. Solve for t.

$$0 = -16t^2 + 8t$$

Substitute 0 for *h*.

$$0=8t(-2t+1)$$

Factor right side.

$$8t = 0 \quad or \quad -2t + 1 = 0$$

Zero-product property

$$t = 0$$
 or $t = \frac{1}{2}$ Solve for t .

$$t=\frac{1}{2}$$

The child lands on the ground $\frac{1}{2}$ second after the child jumps.

Exercise for Example 4

6. In Example 4, suppose the initial velocity is 10 feet per second. After how many seconds will the child land on the ground?

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