8.4 Solve Polynomial Equations in Factored Form

Before You solved linear equations.

Now You will solve polynomial equations.

Why So you can analyze vertical motion, as in Ex. 55.

Key Vocabulary
- roots
- vertical motion model

COMMON CORE

CC.9-12.A.REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

You have learned the property of zero: For any real number a, $a \cdot 0 = 0$. This is equivalent to saying:

For real numbers a and b, if $a = 0$ or $b = 0$, then $ab = 0$.

The converse of this statement is also true (as shown in Exercise 49), and it is called the zero-product property.

KEY CONCEPT *For Your Notebook*

Zero-Product Property

Let a and b be real numbers. If $ab = 0$, then $a = 0$ or $b = 0$.

The zero-product property is used to solve an equation when one side is zero and the other side is a product of polynomial factors. The solutions of such an equation are also called **roots**.

EXAMPLE 1 Use the zero-product property

Solve $(x - 4)(x + 2) = 0$.

$(x - 4)(x + 2) = 0$ **Write original equation.**

$x - 4 = 0$ *or* $x + 2 = 0$ **Zero-product property**

$x = 4$ *or* $x = -2$ **Solve for x.**

▶ The solutions of the equation are 4 and −2.

CHECK Substitute each solution into the original equation to check.

$(4 - 4)(4 + 2) \stackrel{?}{=} 0$ $\quad$ $(-2 - 4)(-2 + 2) \stackrel{?}{=} 0$

$0 \cdot 6 \stackrel{?}{=} 0$ $\quad$ $-6 \cdot 0 \stackrel{?}{=} 0$

$0 = 0$ ✓ $\quad$ $0 = 0$ ✓

GUIDED PRACTICE for Example 1

1. Solve the equation $(x - 5)(x - 1) = 0$.

REVIEW GCF
For help with finding the GCF, see p. SR2.

FACTORING To solve a polynomial equation using the zero-product property, you may need to *factor* the polynomial, or write it as a product of other polynomials. Look for the *greatest common factor* (GCF) of the polynomial's terms. This is a monomial with an integer coefficient that divides evenly into each term.

EXAMPLE 2 Find the greatest common monomial factor

Factor out the greatest common monomial factor.

a. $12x + 42y$ **b.** $4x^4 + 24x^3$

Solution

a. The GCF of 12 and 42 is 6. The variables x and y have no common factor. So, the greatest common monomial factor of the terms is 6.

▶ $12x + 42y = 6(2x + 7y)$

b. The GCF of 4 and 24 is 4. The GCF of x^4 and x^3 is x^3. So, the greatest common monomial factor of the terms is $4x^3$.

▶ $4x^4 + 24x^3 = 4x^3(x + 6)$

✓ GUIDED PRACTICE for Example 2

2. Factor out the greatest common monomial factor from $14m + 35n$.

EXAMPLE 3 Solve an equation by factoring

Solve $2x^2 + 8x = 0$.

$2x^2 + 8x = 0$	Write original equation.
$2x(x + 4) = 0$	Factor left side.
$2x = 0$ *or* $x + 4 = 0$	Zero-product property
$x = 0$ *or* $x = -4$	Solve for x.

▶ The solutions of the equation are 0 and -4.

EXAMPLE 4 Solve an equation by factoring

Solve $6n^2 = 15n$.

AVOID ERRORS
To use the zero-product property, you must write the equation so that one side is 0. For this reason, $15n$ must be subtracted from each side.

$6n^2 - 15n = 0$	Subtract $15n$ from each side.
$3n(2n - 5) = 0$	Factor left side.
$3n = 0$ *or* $2n - 5 = 0$	Zero-product property
$n = 0$ *or* $n = \frac{5}{2}$	Solve for n.

▶ The solutions of the equation are 0 and $\frac{5}{2}$.

GUIDED PRACTICE for Examples 3 and 4

Solve the equation.

3. $a^2 + 5a = 0$ **4.** $3s^2 - 9s = 0$ **5.** $4x^2 = 2x$

VERTICAL MOTION A *projectile* is an object that is propelled into the air but has no power to keep itself in the air. A thrown ball is a projectile, but an airplane is not. The height of a projectile can be described by the **vertical motion model**.

KEY CONCEPT *For Your Notebook*

Vertical Motion Model

The height h (in feet) of a projectile can be modeled by

$$h = -16t^2 + vt + s$$

where t is the time (in seconds) the object has been in the air, v is the initial vertical velocity (in feet per second), and s is the initial height (in feet).

UNDERSTAND THE MODEL
The vertical motion model takes into account the effect of gravity but ignores other, less significant, factors such as air resistance.

EXAMPLE 5 Solve a multi-step problem

ARMADILLO A startled armadillo jumps straight into the air with an initial vertical velocity of 14 feet per second. After how many seconds does it land on the ground?

Solution

STEP 1 **Write** a model for the armadillo's height above the ground.

$h = -16t^2 + vt + s$ **Vertical motion model**

$h = -16t^2 + 14t + 0$ **Substitute 14 for v and 0 for s.**

$h = -16t^2 + 14t$ **Simplify.**

STEP 2 **Substitute** 0 for h. When the armadillo lands, its height above the ground is 0 feet. Solve for t.

$0 = -16t^2 + 14t$ **Substitute 0 for h.**

$0 = 2t(-8t + 7)$ **Factor right side.**

$2t = 0$ *or* $-8t + 7 = 0$ **Zero-product property**

$t = 0$ *or* $t = 0.875$ **Solve for t.**

▶ The armadillo lands on the ground 0.875 second after the armadillo jumps.

AVOID ERRORS
The solution $t = 0$ means that before the armadillo jumps, its height above the ground is 0 feet.

GUIDED PRACTICE for Example 5

6. WHAT IF? In Example 5, suppose the initial vertical velocity is 12 feet per second. After how many seconds does the armadillo land on the ground?

8.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 3 and 55

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 39, 53, and 56

◆ = **MULTIPLE REPRESENTATIONS** Ex. 58

SKILL PRACTICE

1. VOCABULARY What is the vertical motion model and what does each variable in the model represent?

2. ★ WRITING *Explain* how to use the zero-product property to find the solutions of the equation $3x(x - 7) = 0$.

EXAMPLE 1 for Exs. 3–16

ZERO-PRODUCT PROPERTY **Solve the equation.**

3. $(x - 5)(x + 3) = 0$ **4.** $(y + 9)(y - 1) = 0$ **5.** $(z - 13)(z - 14) = 0$

6. $(c + 6)(c + 8) = 0$ **7.** $(d - 7)\left(d + \frac{4}{3}\right) = 0$ **8.** $\left(g - \frac{1}{8}\right)(g + 18) = 0$

9. $(m - 3)(4m + 12) = 0$ **10.** $(2n - 14)(3n + 9) = 0$ **11.** $(3n + 11)(n + 1) = 0$

12. $(3x + 1)(x + 6) = 0$ **13.** $(2y + 5)(7y - 5) = 0$ **14.** $(8z - 6)(12z + 14) = 0$

15. ★ MULTIPLE CHOICE What are the roots of the equation $(y - 12)(y + 6) = 0$?

Ⓐ -12 and -6 Ⓑ -12 and 6 Ⓒ -6 and 12 Ⓓ 6 and 12

16. ERROR ANALYSIS *Describe* and correct the error in solving $(z - 15)(z + 21) = 0$.

$(z - 15)(z + 21) = 0$
$z = -15$ or $z = 21$

EXAMPLE 2 for Exs. 17–26

FACTORING EXPRESSIONS **Factor out the greatest common monomial factor.**

17. $2x + 2y$ **18.** $6x^2 - 15y$ **19.** $3s^4 + 16s$

20. $5d^6 + 2d^5$ **21.** $7w^5 - 35w^2$ **22.** $9m^7 - 3m^2$

23. $15n^3 + 25n$ **24.** $12a^5 + 8a$ **25.** $\frac{5}{2}x^6 - \frac{1}{2}x^4$

26. ERROR ANALYSIS *Describe* and correct the error in factoring out the greatest common monomial factor of $18x^8 - 9x^4 - 6x^3$.

$18x^8 - 9x^4 - 6x^3 = 3x(6x^7 - 3x^3 - 2x^2)$

EXAMPLES 3 and 4 for Exs. 27–39

SOLVING EQUATIONS **Solve the equation.**

27. $b^2 + 6b = 0$ **28.** $5w^2 - 5w = 0$ **29.** $-10n^2 + 35n = 0$

30. $2x^2 + 15x = 0$ **31.** $18c^2 + 6c = 0$ **32.** $-32y^2 - 24y = 0$

33. $3k^2 = 6k$ **34.** $6h^2 = 3h$ **35.** $4s^2 = 10s$

36. $-42z^2 = 14z$ **37.** $28m^2 = -8m$ **38.** $-12p^2 = -30p$

39. ★ MULTIPLE CHOICE What are the solutions of $4x^2 = x$?

Ⓐ -4 and 0 Ⓑ $-\frac{1}{4}$ and 0 Ⓒ 0 and $\frac{1}{4}$ Ⓓ 0 and 4

FACTORING EXPRESSIONS **Factor out the greatest common monomial factor.**

40. $20x^2y^2 - 4xy$ **41.** $8a^2b - 6ab^2$ **42.** $18s^2t^5 - 2s^3t$

43. $v^3 - 5v^2 + 9v$ **44.** $-2g^4 + 14g^2 + 6g$ **45.** $6q^5 - 21q^4 - 15q^2$

HINT
You may want to review finding zeros of linear functions before finding zeros of quadratic functions.

FINDING ZEROS OF FUNCTIONS **Find the zeros of the function.**

46. $f(x) = x^2 - 15x$ **47.** $f(x) = -2x^2 + x$ **48.** $f(x) = 3x^2 - 27x$

49. **CHALLENGE** Consider the equation $ab = 0$. Assume that $a \neq 0$ and solve the equation for b. Then assume that $b \neq 0$ and solve the equation for a. What conclusion can you draw about the values of a and b?

50. **CHALLENGE** Consider the equation $z = x^2 - xy$. For what values of x and y does $z = 0$?

PROBLEM SOLVING

EXAMPLE 5
for Exs. 51–53

51. **MOTION** A cat leaps from the ground into the air with an initial vertical velocity of 11 feet per second. After how many seconds does the cat land on the ground?

52. **SPITTLEBUG** A spittlebug jumps into the air with an initial vertical velocity of 10 feet per second.

a. Write an equation that gives the height of the spittlebug as a function of the time (in seconds) since it left the ground.

b. The spittlebug reaches its maximum height after 0.3125 second. How high can it jump?

53. ★ **SHORT RESPONSE** A penguin jumps out of the water while swimming. This action is called porpoising. The height h (in feet) of the porpoising penguin can be modeled by $h = -16t^2 + 4.5t$ where t is the time (in seconds) since the penguin jumped out of the water. Find the zeros of the function. *Explain* what the zeros mean in this situation.

VERTICAL MOTION **In Exercises 54 and 55, use the information below.**

The height h (in meters) of a projectile can be modeled by $h = -4.9t^2 + vt + s$ where t is the time (in seconds) the object has been in the air, v is the initial vertical velocity (in meters per second), and s is the initial height (in meters).

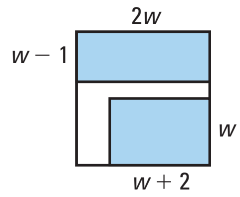
54. **SOCCER** A soccer ball is kicked upward from the ground with an initial vertical velocity of 3.6 meters per second. After how many seconds does it land?

55. **RABBIT HIGH JUMP** A rabbit in a high jump competition leaves the ground with an initial vertical velocity of 4.9 meters per second.

a. Write an equation that gives the height of the rabbit as a function of the time (in seconds) since it left the ground.

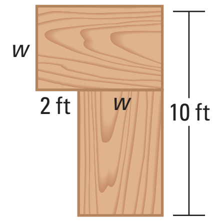
b. What is a reasonable domain for the function? *Explain* your answer.

56. ★ **MULTIPLE CHOICE** Two rectangular rooms in a building's floor plan have different dimensions but the same area. The dimensions (in meters) are shown. What is the value of w?



Ⓐ 3 m   Ⓑ 4 m   Ⓒ 6 m   Ⓓ 8 m

57. **TABLETOP AREAS** A display in your school library sits on top of two rectangular tables arranged in an L shape, as shown. The tabletops have the same area.



a. Write an equation that relates the areas of the tabletops.

b. Find the value of w.

c. What is the combined area of the tabletops?

58. **MULTIPLE REPRESENTATIONS** An arch frames the entrance to a garden. The shape of the arch is modeled by the graph of the equation $y = -2x^2 + 8x$ where x and y are measured in feet. On a coordinate plane, the ground is represented by the x-axis.

a. **Making a Table** Make a table of values that shows the height of the arch for $x = 0, 1, 2, 3,$ and 4 feet.

b. **Drawing a Graph** Plot the ordered pairs in the table as points in a coordinate plane. Connect the points with a smooth curve that represents the arch.

c. **Interpreting a Graph** How wide is the base of the arch?

59. **CHALLENGE** The shape of an arched doorway is modeled by the graph of the function $y = -0.5x(x - 8)$ where x and y are measured in feet. On a coordinate plane, the floor is represented by the x-axis.

a. How wide is the doorway at its base? *Justify* your answer using the zeros of the function.

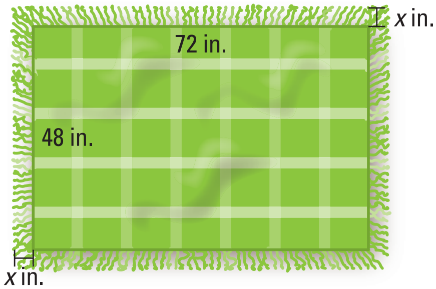
b. The doorway's highest point occurs above the center of its base. How high is the highest point of the arched doorway? *Explain* how you found your answer.

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

MIXED REVIEW *of Problem Solving*

MATHEMATICAL PRACTICES Make sense of problems and persevere in solving them.

1. **MULTI-STEP PROBLEM** You are making a blanket with a fringe border of equal width on each edge, as shown.



   a. Write a polynomial that represents the total area of the blanket with the fringe.

   b. Find the total area of the blanket with fringe when the width of the fringe is 4 inches.

2. **OPEN-ENDED** A horse with pinto coloring has white fur with patches of color. The gene P is for pinto coloring, and the gene s is for solid coloring. Any gene combination with a P results in pinto coloring.

   a. Suppose a male horse has the gene combination Ps. Choose a color gene combination for a female horse. Create a Punnett square to show the possible gene combinations of the two horses' offspring.

   b. What percent of the possible gene combinations of the offspring result in pinto coloring?

   c. Show how you could use a polynomial to model the possible color gene combinations of the offspring.

3. **SHORT RESPONSE** One football is kicked into the air with an initial vertical velocity of 44 feet per second. Another football is kicked into the air with an initial vertical velocity of 40 feet per second.

   a. Which football is in the air for more time?

   b. *Justify* your answer to part (a).

4. **GRIDDED ANSWER** During the period 1996–2000, the total value T (in millions of dollars) of toys imported to the United States can be modeled by

   $$T = 82.9t^3 - 848t^2 + 3030t + 9610$$

   where t is the number of years since 1996. What is the degree of the polynomial that represents T?

5. **EXTENDED RESPONSE** During the period 1992–2000, the number C (in millions) of people participating in cross-country skiing and the number S (in millions) of people participating in snowboarding can be modeled by

   $$C = 0.067t^3 - 0.107t^2 + 0.27t + 3.5$$

   $$S = 0.416t + 1.24$$

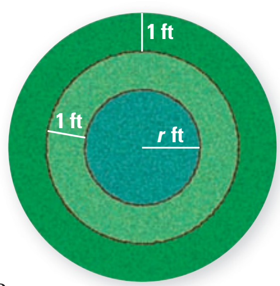
   where t is the number of years since 1992.

   a. Write an equation that models the total number of people T (in millions) participating in cross-country skiing and snowboarding as a function of the number of years since 1992.

   b. Find the total participation in these activities in 1992 and 2000.

   c. What was the average rate of change in total participation from 1992 to 2000? *Explain* how you found this rate.

6. **SHORT RESPONSE** A circular rug has an interior circle and two rings around the circle, as shown.



   a. Write a polynomial that represents the total area of the rug. Leave your answer in terms of π.

   b. The interior circle of the rug has a diameter of 3 feet. What is the area of the rug? Leave your answer in terms of π. *Explain* how you found your answer.