8.5 Factor $x^2 + bx + c$

Before You factored out the greatest common monomial factor.

Now You will factor trinomials of the form $x^2 + bx + c$.

Why So you can find the dimensions of figures, as in Ex. 61.

Key Vocabulary
- **zero of a function**

You know from multiplying binomials that

$$(x + 3)(x + 4) = x^2 + (4 + 3)x + 4 \cdot 3 = x^2 + 7x + 12.$$

You will reverse this process to factor trinomials of the form $x^2 + bx + c$.

COMMON CORE

CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

KEY CONCEPT *For Your Notebook*

Factoring $x^2 + bx + c$

Algebra $x^2 + bx + c = (x + p)(x + q)$ provided $p + q = b$ and $pq = c$.

Example $x^2 + 5x + 6 = (x + 3)(x + 2)$ because $3 + 2 = 5$ and $3 \cdot 2 = 6$.

EXAMPLE 1 Factor when b and c are positive

Factor $x^2 + 11x + 18$.

Solution

Find two positive factors of 18 whose sum is 11. Make an organized list.

Factors of 18	Sum of factors	
18, 1	$18 + 1 = 19$	✗
9, 2	$9 + 2 = 11$	← Correct sum
6, 3	$6 + 3 = 9$	✗

The factors 9 and 2 have a sum of 11, so they are the correct values of p and q.

▶ $x^2 + 11x + 18 = (x + 9)(x + 2)$

CHECK $(x + 9)(x + 2) = x^2 + 2x + 9x + 18$ Multiply binomials.

$= x^2 + 11x + 18$ ✓ Simplify.

GUIDED PRACTICE for Example 1

Factor the trinomial.

1. $x^2 + 3x + 2$ **2.** $a^2 + 7a + 10$ **3.** $t^2 + 9t + 14$

FACTORING When factoring a trinomial, first consider the signs of p and q.

$(x + p)(x + q)$	$x^2 + bx + c$	Signs of b and c
$(x + 2)(x + 3)$	$x^2 + 5x + 6$	b is positive; c is positive.
$(x + 2)(x + (-3))$	$x^2 - x - 6$	b is negative; c is negative.
$(x + (-2))(x + 3)$	$x^2 + x - 6$	b is positive; c is negative.
$(x + (-2))(x + (-3))$	$x^2 - 5x + 6$	b is negative; c is positive.

By observing the signs of b and c in the table, you can see that:

- b and c are positive when both p and q are positive.
- b is negative and c is positive when both p and q are negative.
- c is negative when p and q have different signs.

EXAMPLE 2 Factor when b is negative and c is positive

Factor $n^2 - 6n + 8$.

Because b is negative and c is positive, p and q must both be negative.

Factors of 8	Sum of factors	
$-8, -1$	$-8 + (-1) = -9$	✗
$-4, -2$	$-4 + (-2) = -6$	← Correct sum

▶ $n^2 - 6n + 8 = (n - 4)(n - 2)$

EXAMPLE 3 Factor when b is positive and c is negative

Factor $y^2 + 2y - 15$.

Because c is negative, p and q must have different signs.

Factors of −15	Sum of factors	
$-15, 1$	$-15 + 1 = -14$	✗
$15, -1$	$15 + (-1) = 14$	✗
$-5, 3$	$-5 + 3 = -2$	✗
$5, -3$	$5 + (-3) = 2$	← Correct sum

▶ $y^2 + 2y - 15 = (y + 5)(y - 3)$

GUIDED PRACTICE for Examples 2 and 3

Factor the trinomial.

4. $x^2 - 4x + 3$ **5.** $t^2 - 8t + 12$ **6.** $m^2 + m - 20$ **7.** $w^2 + 6w - 16$

EXAMPLE 4 Solve a polynomial equation

Solve the equation $x^2 + 3x = 18$.

$x^2 + 3x = 18$		Write original equation.
$x^2 + 3x - 18 = 0$		Subtract 18 from each side.
$(x + 6)(x - 3) = 0$		Factor left side.
$x + 6 = 0$	*or* $x - 3 = 0$	Zero-product property
$x = -6$	*or* $x = 3$	Solve for x.

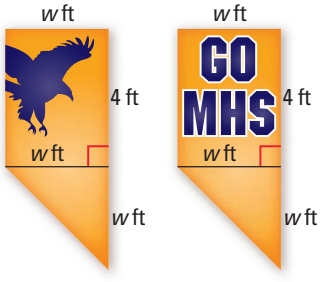
▶ The solutions of the equation are -6 and 3.

GUIDED PRACTICE for Example 4

8. Solve the equation $s^2 - 2s = 24$.

EXAMPLE 5 Solve a multi-step problem

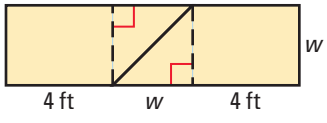
BANNER DIMENSIONS You are making banners to hang during school spirit week. Each banner requires 16.5 square feet of felt and will be cut as shown. Find the width of one banner.



ANOTHER WAY
For alternative methods for solving Example 5, see the **Problem Solving Workshop**.

Solution

STEP 1 **Draw** a diagram of two banners together.



STEP 2 **Write** an equation using the fact that the area of 2 banners is $2(16.5) = 33$ square feet. Solve the equation for w.

$A = \ell \cdot w$		Formula for area of a rectangle
$33 = (4 + w + 4) \cdot w$		Substitute 33 for A and $(4 + w + 4)$ for ℓ.
$0 = w^2 + 8w - 33$		Simplify and subtract 33 from each side.
$0 = (w + 11)(w - 3)$		Factor right side.
$w + 11 = 0$	*or* $w - 3 = 0$	Zero-product property
$w = -11$	*or* $w = 3$	Solve for w.

▶ The banner cannot have a negative width, so the width is 3 feet.

GUIDED PRACTICE for Example 5

9. WHAT IF? In Example 5, suppose the area of a banner is to be 10 square feet. What is the width of one banner?

8.5 EXERCISES

HOMEWORK KEY

- ◯ = See **WORKED-OUT SOLUTIONS** Exs. 7 and 61
- ★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 29, 42, 61, 62, and 63
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 64

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The ___?___ of $t^2 + 3t + 2$ are $t + 2$ and $t + 1$.

2. ★ **WRITING** If $x^2 - 8x + 12 = (x + p)(x + q)$, what are the signs of p and q? *Justify* your answer.

EXAMPLES 1, 2, and 3 for Exs. 3–19

FACTORING TRINOMIALS **Factor the trinomial.**

3. $x^2 + 4x + 3$
4. $a^2 + 6a + 8$
5. $b^2 - 17b + 72$
6. $s^2 - 10s + 16$
7. $z^2 + 8z - 48$
8. $w^2 + 18w + 56$
9. $y^2 - 7y - 18$
10. $n^2 - 9n + 14$
11. $x^2 + 3x - 70$
12. $f^2 + 4f - 32$
13. $m^2 - 7m - 120$
14. $d^2 - 20d + 99$
15. $p^2 + 20p + 64$
16. $x^2 + 6x - 72$
17. $c^2 + 15c + 44$

ERROR ANALYSIS ***Describe*** **and correct the error in factoring the trinomial.**

18. $s^2 - 17s - 60 = (s - 5)(s - 12)$ ✗

19. $m^2 - 10m + 24 = (m - 12)(m + 2)$ ✗

EXAMPLE 4 for Exs. 20–29

SOLVING EQUATIONS **Solve the equation.**

20. $x^2 - 10x + 21 = 0$
21. $n^2 - 7n - 30 = 0$
22. $w^2 - 15w + 44 = 0$
23. $a^2 + 5a = 50$
24. $r^2 + 2r = 24$
25. $t^2 + 9t = -20$
26. $y^2 - 2y - 8 = 7$
27. $m^2 + 22 = -23m$
28. $b^2 + 5 = 8b - 10$

29. ★ **MULTIPLE CHOICE** What are the solutions of the equation $x^2 - 8x = 240$?

Ⓐ -20 and -12 Ⓑ -20 and 12

Ⓒ 20 and -12 Ⓓ 12 and 20

FINDING ZEROS OF FUNCTIONS **Find the zeros of the polynomial function.**

30. $f(x) = x^2 + 11x + 18$
31. $g(x) = x^2 + 5x + 6$
32. $h(x) = x^2 - 18x + 32$
33. $f(x) = x^2 - 14x + 45$
34. $h(x) = x^2 - 5x - 24$
35. $g(x) = x^2 - 14x - 51$
36. $g(x) = x^2 + 10x - 39$
37. $f(x) = -x^2 + 16x - 28$
38. $f(x) = -x^2 + 24x + 180$

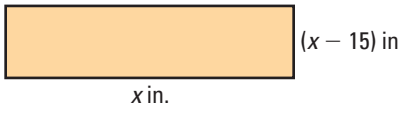
SOLVING EQUATIONS **Solve the equation.**

39. $s(s + 1) = 72$
40. $x^2 - 10(x - 1) = -11$
41. $q(q + 19) = -34$

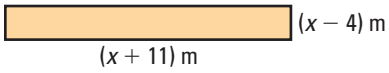
42. ★ **SHORT RESPONSE** Write an equation of the form $x^2 + bx + c = 0$ that has the solutions -4 and 6. *Explain* how you found your answer.

GEOMETRY **Find the dimensions of the rectangle or triangle that has the given area.**

43. Area: 100 square inches

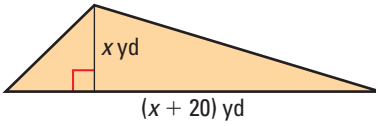


44. Area: 34 square meters

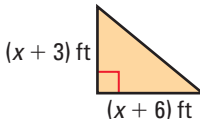


HINT
In Ex. 45, convert the given area to square yards. Use the conversion factor $\frac{1 \text{ yd}^2}{9 \text{ ft}^2}$.

45. Area: 702 square feet



46. Area: 119 square feet



FACTORING TRINOMIALS **In Exercises 47–55, use the example below to factor the trinomial.**

EXAMPLE **Factor a trinomial in two variables**

Factor $x^2 + 9xy + 14y^2$.

Solution

To factor the trinomial, you must find factors of the form $x + py$ and $x + qy$.

First, consider the signs of the factors needed. In this example, b is 9, and c is 14. Because both b and c are positive, you must find two positive factors of 14 that have a sum of 9.

Factors of 14	Sum of factors	
14, 1	14 + 1 = 15	✗
7, 2	7 + 2 = 9	← Correct sum

The factors 7 and 2 have a sum of 9, so 7 and 2 are the correct values of p and q.

▶ $x^2 + 9xy + 14y^2 = (x + 7y)(x + 2y)$

47. $x^2 - 4xy + 4y^2$ **48.** $y^2 - 6yz + 5z^2$ **49.** $c^2 + 13cd + 36d^2$

50. $r^2 + 15rs + 50s^2$ **51.** $a^2 + 2ab - 15b^2$ **52.** $x^2 + 8xy - 65y^2$

53. $m^2 - mn - 42n^2$ **54.** $u^2 - 3uv - 108v^2$ **55.** $g^2 + 4gh - 60h^2$

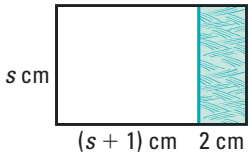
CHALLENGE **Find all integer values of b for which the trinomial has factors of the form $x + p$ and $x + q$ where p and q are integers.**

56. $x^2 + bx + 15$ **57.** $x^2 - bx + 21$ **58.** $x^2 + bx - 42$

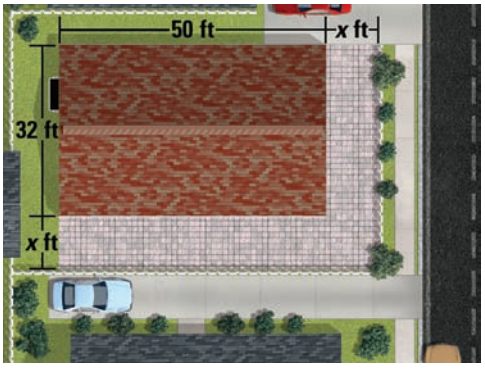
PROBLEM SOLVING

EXAMPLE 5
for Exs. 59–61

59. CARD DESIGN You are designing a gift card that has a border along one side, as shown. The area of the white part of the card is 30 square centimeters. What is the area of the border?



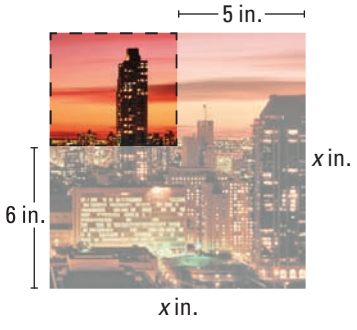
60. CONSTRUCTION A contractor is building a porch along two sides of a house. The house is rectangular with a width of 32 feet and a length of 50 feet. The porch will have the same width on each side of the house.



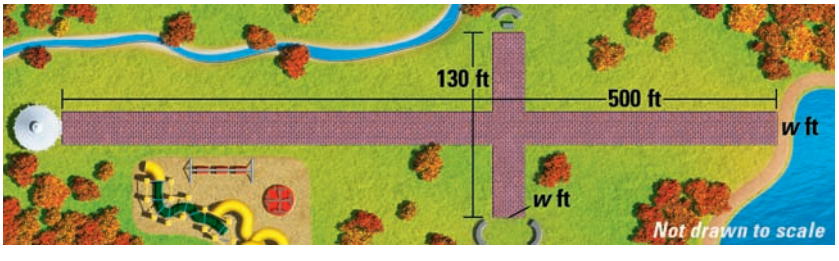
a. Write a polynomial that represents the combined area of the first floor of the house and the porch.

b. The owners want the combined area of the first floor and the porch to be 2320 square feet. How wide should the contractor build the porch?

61. ★ SHORT RESPONSE You trimmed a large square picture so that you could fit it into a frame. You trimmed 6 inches from the length and 5 inches from the width. The area of the resulting picture is 20 square inches. What was the perimeter of the original large square picture? *Explain* how you found your answer.



62. ★ EXTENDED RESPONSE A town has a rectangular park. The parks department is planning to install two brick paths that will intersect at right angles. One path will be 130 feet long, and the other path will be 500 feet long. The paths will have the same width.



HINT
Add the path areas, but subtract the overlap, so that it is not counted twice.

a. Write a polynomial that represents the combined area of the two paths.

b. The parks department can afford brick for 3125 square feet of path. Write and solve an equation to find the width of the paths.

c. In part (b) you used one solution of the equation to find your answer. *Explain* how you chose which solution to use.

◯ = See WORKED-OUT SOLUTIONS in Student Resources ★ = STANDARDIZED TEST PRACTICE = MULTIPLE REPRESENTATIONS

63. ★ **MULTIPLE CHOICE** A square quilt has a border that is 1 foot wide on each side. The quilt has an area of 25 square feet. What is the side length of the quilt without the border?

Ⓐ 2 feet Ⓑ 3 feet Ⓒ 4 feet Ⓓ 5 feet

64. **MULTIPLE REPRESENTATIONS** You toss a set of keys to a friend who is standing at a window 20 feet above the ground in a building that is 5 feet away from where you are standing. The path of the keys can be modeled by the graph of the equation $y = -x^2 + 8x + 5$ where x and y are measured in feet. On a coordinate plane, the ground is represented by the x-axis, and you are standing at the origin.

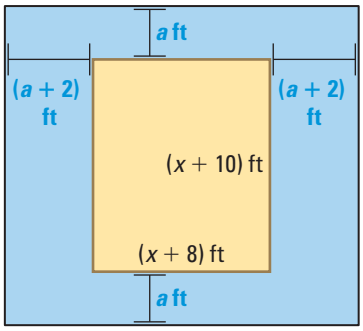
a. Making a Table Make a table of values that shows the height of the keys for $x = 2$, 4, 6, and 8 feet.

b. Drawing a Graph Plot the ordered pairs in the table as points in a coordinate plane. Connect the points with a smooth curve.

c. Interpreting a Graph Based on your graph, do you expect the keys to reach your friend? *Explain* your answer.

d. Using an Equation Find the value of x when $y = 20$. (You may need to factor out a -1 in order to factor the trinomial.) What do you notice? *Explain* how the x-value justifies your answer from part (c).

65. **CHALLENGE** A rectangular stage is positioned in the center of a rectangular room, as shown. The area of the stage is 120 square feet.



a. Use the dimensions given in the diagram to find the length and width of the stage.

b. The combined area of the stage and the surrounding floor is 360 square feet. Find the length and width of the room.

PROBLEM SOLVING WORKSHOP
LESSON 8.5

Using ALTERNATIVE METHODS

Another Way to Solve Example 5

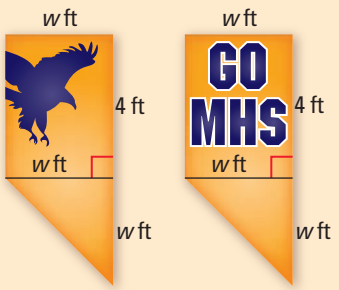
MATHEMATICAL PRACTICES

Make sense of problems and persevere in solving them.

MULTIPLE REPRESENTATIONS In Example 5, you saw how to solve the problem about a school banner by solving an equation. You can also solve the problem using a table or a graph.

PROBLEM

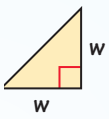
BANNER DIMENSIONS You are making banners to hang during school spirit week. Each banner requires 16.5 square feet of felt and will be cut as shown. Find the width of one banner.



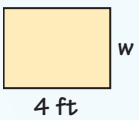
METHOD 1

Using a Table Consider the separate geometric figures that form one banner and find their areas in terms of w. Then find the total area of the banner for different values of w until you find a value that gives a total area of 16.5 square feet. Use a table to organize your work.

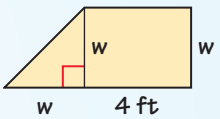
STEP 1 **Write** equations for the area of the pieces and the total area.



$A = \frac{1}{2}w^2$



$A = 4w$



$A = \frac{1}{2}w^2 + 4w$

STEP 2 **Organize** your work in a table.

w	Triangle's area $\left(\frac{1}{2}w^2\right)$	Rectangle's area $(4w)$	Total area $\left(\frac{1}{2}w^2 + 4w\right)$	
1	0.5	4	4.5	← 4.5 < 16.5, so try a greater value of w.
2	2	8	10	← 10 < 16.5, so try a greater value of w.
3	4.5	12	16.5	← Correct area

▶ The width of the banner is 3 feet.

METHOD 2

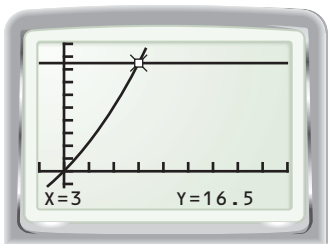
Using a Graph Another approach is to use a graph.

STEP 1 **Write** an equation for the area of the banner. The area of the banner can be thought of as the area of a triangle plus the area of a rectangle.

Area of banner = Area of triangle + Area of rectangle

$$A = \frac{1}{2}w^2 + 4w$$

STEP 2 **Graph** the equation for the area of the banner using a graphing calculator. Graph $y_1 = 0.5x^2 + 4x$. Because you are looking for the value of x that gives an area of 16.5 square feet, you should display the graph of $y_2 = 16.5$ in the same viewing window.

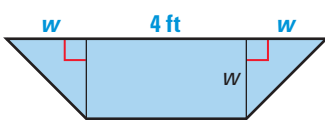


STEP 3 **Find** the intersection of the graphs by using the *intersect* feature on your calculator. The graphs intersect at (3, 16.5).

▶ The width of the banner is 3 feet.

PRACTICE

1. **COUNTER DESIGN** A contractor is building a counter in a kitchen using the diagram shown. The countertop will have an area of 12 square feet. How wide should it be? Solve this problem using two different methods.

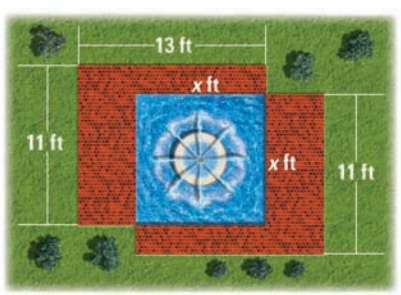


2. **ERROR ANALYSIS** *Describe* and correct the error in using an equation to solve the problem in Exercise 1.

$$12 = 4w + \frac{1}{2}w^2 + \frac{1}{2}w^2$$
$$0 = w^2 + 4w - 12$$
$$0 = (w + 2)(w - 6)$$
$$w + 2 = 0 \quad \text{or} \quad w - 6 = 0$$
$$w = -2 \quad \text{or} \quad w = 6$$

The width is 6 feet.

3. **FOUNTAIN DESIGN** A square fountain in a city plaza is surrounded by brick patios as shown. The combined area of the fountain and brick patios is 205 square feet. What is the side length of the fountain? Solve this problem using two different methods.



4. **WHAT IF?** You want to make a larger banner using the same pattern shown in the problem. The new banner will have an area of 24 square feet. Find the width of the new banner. *Describe* the method you used to find your answer.