8.7 Factor Special Products

Before You factored polynomials of the form $ax^2 + bx + c$.

Now You will factor special products.

Why? So you can use a scientific model, as in Ex. 48.

Key Vocabulary
- **perfect square trinomial**

You can use the special product patterns you have learned to factor polynomials, such as the difference of two squares.

KEY CONCEPT *For Your Notebook*

Difference of Two Squares Pattern

Algebra	Example
$a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$

COMMON CORE

CC.9-12.A.SSE.2 Use the struture of an expression to identify ways to rewrite it.

EXAMPLE 1 Factor the difference of two squares

Factor the polynomial.

a. $y^2 - 16 = y^2 - 4^2$ — Write as $a^2 - b^2$.

$= (y + 4)(y - 4)$ — Difference of two squares pattern

b. $25m^2 - 36 = (5m)^2 - 6^2$ — Write as $a^2 - b^2$.

$= (5m + 6)(5m - 6)$ — Difference of two squares pattern

c. $x^2 - 49y^2 = x^2 - (7y)^2$ — Write as $a^2 - b^2$.

$= (x + 7y)(x - 7y)$ — Difference of two squares pattern

EXAMPLE 2 Factor the difference of two squares

Factor the polynomial $8 - 18n^2$.

$8 - 18n^2 = 2(4 - 9n^2)$ — Factor out common factor.

$= 2[2^2 - (3n)^2]$ — Write $4 - 9n^2$ as $a^2 - b^2$.

$= 2(2 + 3n)(2 - 3n)$ — Difference of two squares pattern

GUIDED PRACTICE for Examples 1 and 2

1. Factor the polynomial $4y^2 - 64$.

PERFECT SQUARE TRINOMIALS The pattern for finding the square of a binomial gives you the pattern for factoring trinomials of the form $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$. These are called **perfect square trinomials**.

KEY CONCEPT *For Your Notebook*

Perfect Square Trinomial Pattern

Algebra	Example
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 6x + 9 = x^2 + 2(x \cdot 3) + 3^2 = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 10x + 25 = x^2 - 2(x \cdot 5) + 5^2 = (x - 5)^2$

EXAMPLE 3 Factor perfect square trinomials

Factor the polynomial.

a. $n^2 - 12n + 36 = n^2 - 2(n \cdot 6) + 6^2$ — Write as $a^2 - 2ab + b^2$.

$= (n - 6)^2$ — Perfect square trinomial pattern

b. $9x^2 - 12x + 4 = (3x)^2 - 2(3x \cdot 2) + 2^2$ — Write as $a^2 - 2ab + b^2$.

$= (3x - 2)^2$ — Perfect square trinomial pattern

c. $4s^2 + 4st + t^2 = (2s)^2 + 2(2s \cdot t) + t^2$ — Write as $a^2 + 2ab + b^2$.

$= (2s + t)^2$ — Perfect square trinomial pattern

Animated Algebra at my.hrw.com

EXAMPLE 4 Factor a perfect square trinomial

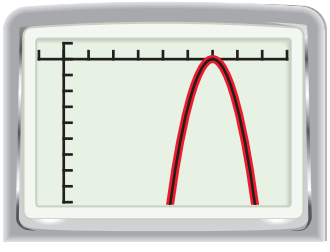
Factor the polynomial $-3y^2 + 36y - 108$.

$-3y^2 + 36y - 108 = -3(y^2 - 12y + 36)$ — Factor out -3.

$= -3[y^2 - 2(y \cdot 6) + 6^2]$ — Write $y^2 - 12y + 36$ as $a^2 - 2ab + b^2$.

$= -3(y - 6)^2$ — Perfect square trinomial pattern

CHECK Check your factorization using a graphing calculator. Graph $y_1 = -3x^2 + 36x - 108$ and $y_2 = -3(x - 6)^2$. Because the graphs coincide, you know that your factorization is correct.



GUIDED PRACTICE for Examples 3 and 4

Factor the polynomial.

2. $h^2 + 4h + 4$ **3.** $2y^2 - 20y + 50$ **4.** $3x^2 + 6xy + 3y^2$

EXAMPLE 5 Solve a polynomial equation

Solve the equation $x^2 + \frac{2}{3}x + \frac{1}{9} = 0$.

FIND SOLUTIONS
This equation has two identical solutions, because it has two identical factors.

$$x^2 + \frac{2}{3}x + \frac{1}{9} = 0 \qquad \text{Write original equation.}$$

$$9x^2 + 6x + 1 = 0 \qquad \text{Multiply each side by 9.}$$

$$(3x)^2 + 2(3x \cdot 1) + (1)^2 = 0 \qquad \text{Write left side as } a^2 + 2ab + b^2.$$

$$(3x + 1)^2 = 0 \qquad \text{Perfect square trinomial pattern}$$

$$3x + 1 = 0 \qquad \text{Zero-product property}$$

$$x = -\frac{1}{3} \qquad \text{Solve for } x.$$

▸ The solution of the equation is $-\frac{1}{3}$.

EXAMPLE 6 Solve a vertical motion problem

FALLING OBJECT A window washer drops a wet sponge from a height of 64 feet. After how many seconds does the sponge land on the ground?

Solution

Use the vertical motion model to write an equation for the height h (in feet) of the sponge as a function of the time t (in seconds) after it is dropped.

The sponge was dropped, so it has no initial vertical velocity. Find the value of t for which the height is 0.

$$h = -16t^2 + vt + s \qquad \text{Vertical motion model}$$

$$0 = -16t^2 + (0)t + 64 \qquad \text{Substitute 0 for } h\text{, 0 for } v\text{, and 64 for } s.$$

$$0 = -16(t^2 - 4) \qquad \text{Factor out } -16.$$

$$0 = -16(t - 2)(t + 2) \qquad \text{Difference of two squares pattern}$$

$$t - 2 = 0 \quad \text{or} \quad t + 2 = 0 \qquad \text{Zero-product property}$$

$$t = 2 \quad \text{or} \quad t = -2 \qquad \text{Solve for } t.$$

Disregard the negative solution of the equation.

▸ The sponge lands on the ground 2 seconds after it is dropped.

GUIDED PRACTICE for Examples 5 and 6

Solve the equation.

5. $a^2 + 6a + 9 = 0$ **6.** $w^2 - 14w + 49 = 0$ **7.** $n^2 - 81 = 0$

8. WHAT IF? In Example 6, suppose the sponge is dropped from a height of 16 feet. After how many seconds does it land on the ground?

8.7 EXERCISES

HOMEWORK KEY

◯ = See **WORKED-OUT SOLUTIONS** Exs. 11 and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 24, 49, and 50

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: The polynomial $9n^2 + 6n + 1$ is called a(n) __?__ trinomial.

2. ★ **WRITING** *Explain* how to factor the difference of two squares.

EXAMPLES 1 and 2 for Exs. 3–8

DIFFERENCE OF TWO SQUARES **Factor the polynomial.**

3. $x^2 - 25$
4. $n^2 - 64$
5. $81c^2 - 4$
6. $49 - 121p^2$
7. $-3m^2 + 48n^2$
8. $225x^2 - 144y^2$

EXAMPLES 3 and 4 for Exs. 9–14

PERFECT SQUARE TRINOMIALS **Factor the polynomial.**

9. $x^2 - 4x + 4$
10. $y^2 - 10y + 25$
11. $49a^2 + 14a + 1$
12. $9t^2 - 12t + 4$
13. $m^2 + m + \frac{1}{4}$
14. $2x^2 + 12xy + 18y^2$

EXAMPLES 1, 2, 3, and 4 for Exs. 15–24

FACTORING POLYNOMIALS **Factor the polynomial.**

15. $4c^2 - 400$
16. $4f^2 - 36f + 81$
17. $-9r^2 + 4s^2$
18. $z^2 + 12z + 36$
19. $72 - 32y^2$
20. $45r^2 - 120rs + 80s^2$

ERROR ANALYSIS ***Describe*** **and correct the error in factoring.**

21.
$$36x^2 - 81 = 9(4x^2 - 9)$$
$$= 9((2x)^2 - 3^2)$$
$$= 9(2x - 3)^2$$

22.
$$y^2 - 6y + 9 = y^2 - 2(y \cdot 3) + 3^2$$
$$= (y - 3)(y + 3)$$

23. ★ **MULTIPLE CHOICE** Which is the correct factorization of $-45x^2 + 20y^2$?

(A) $-5(3x + 2y)^2$ (B) $5(3x - 2y)^2$

(C) $-5(3x + 2y)(3x - 2y)$ (D) $5(3x + 2y)(3x - 2y)$

24. ★ **MULTIPLE CHOICE** Which is the correct factorization of $16m^2 - 8mn + n^2$?

(A) $(4m - n)^2$ (B) $(4m + n)^2$

(C) $(8m - n)^2$ (D) $(4m - n)(4m + n)$

EXAMPLE 5 for Exs. 25–39

SOLVING EQUATIONS **Solve the equation.**

25. $x^2 + 8x + 16 = 0$
26. $16a^2 - 8a + 1 = 0$
27. $4w^2 - 36 = 0$
28. $32 - 18m^2 = 0$
29. $27c^2 + 108c + 108 = 0$
30. $-2h^2 - 28h - 98 = 0$
31. $6p^2 = 864$
32. $-3t^2 = -108$
33. $8k^2 = 98$
34. $-\frac{4}{3}x + \frac{4}{9} = -x^2$
35. $y^2 - \frac{5}{3}y = -\frac{25}{36}$
36. $\frac{2}{9} = 8n^2$
37. $-9c^2 = -16$
38. $-20s - 3 = 25s^2 + 1$
39. $y^4 - 2y^3 + y^2 = 0$

CHALLENGE **Determine the value(s) of k for which the expression is a perfect square trinomial.**

40. $x^2 + kx + 36$ **41.** $4x^2 + kx + 9$ **42.** $16x^2 + kx + 4$

43. $25x^2 + 10x + k$ **44.** $49x^2 - 84x + k$ **45.** $4x^2 - 48x + k$

PROBLEM SOLVING

EXAMPLE 6 for Exs. 46–48

46. FALLING BRUSH While standing on a ladder, you drop a paintbrush from a height of 25 feet. After how many seconds does the paintbrush land on the ground?

47. FALLING OBJECT A hickory nut falls from a branch that is 100 feet above the ground. After how many seconds does the hickory nut land on the ground?

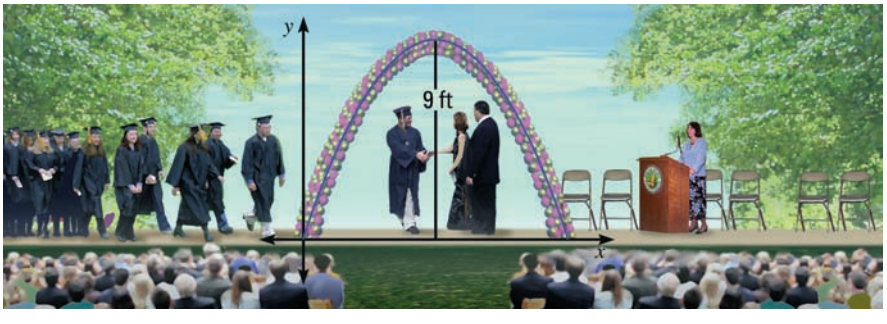
48. GRASSHOPPER A grasshopper jumps straight up from the ground with an initial vertical velocity of 8 feet per second.

a. Write an equation that gives the height (in feet) of the grasshopper as a function of the time (in seconds) since it leaves the ground.

b. After how many seconds is the grasshopper 1 foot off the ground?

49. ★ **SHORT RESPONSE** A ball is thrown up into the air from a height of 5 feet with an initial vertical velocity of 56 feet per second. How many times does the ball reach a height of 54 feet? *Explain* your answer.

50. ★ **EXTENDED RESPONSE** An arch of balloons decorates the stage at a high school graduation. The balloons are tied to a frame. The shape of the frame can be modeled by the graph of the equation $y = -\frac{1}{4}x^2 + 3x$ where x and y are measured in feet.



a. Make a table of values that shows the height of the balloon arch for $x = 0$, 2, 5, 8, and 11 feet.

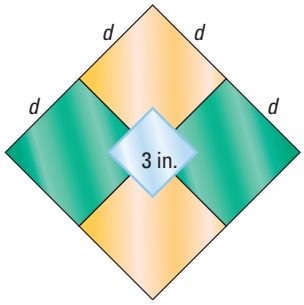
b. For what additional values of x does the equation make sense? *Explain*.

c. At approximately what distance from the left end does the arch reach a height of 9 feet? Check your answer algebraically.

◯ = See WORKED-OUT SOLUTIONS in Student Resources

★ = STANDARDIZED TEST PRACTICE

51. FRAMING A square mirror is framed with stained glass as shown. Each corner of the frame began as a square with a side length of d inches before it was cut to fit the mirror. The mirror has a side length of 3 inches. The area of the stained glass frame is 91 square inches.



a. Write a polynomial that represents the area of the stained glass frame.

b. What is the side length of the frame?

52. CHALLENGE You have 120 folding chairs to set up in a park for an outdoor play. You want each row to have an odd number of chairs. You also want each row after the first to have 2 more chairs than the row in front of it. The first row will have 15 chairs.

a. Copy and complete the table below.

n	nth odd integer	Sum of first n odd integers	Sum as a power
1	1	1	1^2
2	3	$1 + 3 = 4$	2^2
3	5	$1 + 3 + 5 = 9$	?
4	7	?	?
5	9	?	?

b. *Describe* the relationship between n and the sum of the first n odd integers. Then find the sum of the first 10 odd integers.

c. *Explain* how to find the sum of the odd integers from 11 to 21.

d. How many rows of chairs will you need for the outdoor play? *Explain* your thinking.