LESSON 8.8 | **Study Guide**
For use with the lesson "Factor Polynomials Completely"

GOAL | **Factor polynomials completely.**

> ### Vocabulary
>
> Factoring a common monomial from pairs of terms, then looking for a common binomial factor is called **factor by grouping.**
>
> A polynomial of two or more terms is **prime** if it cannot be written as the product of polynomials of lesser degree using only integer coefficients and constants, and if the only common factors of its terms are 1 and –1.
>
> A polynomial is factored completely if it is written as a monomial or as the product of a monomial (possibly 1 or −1) and one or more prime polynomials.

EXAMPLE 1 | **Factor out a common binomial**

Factor the expression.

   a. $5x^2(x - 2) - 3(x - 2)$ **b.** $7y(5 - y) + 3(y - 5)$

Solution

   a. $5x^2(x - 2) - 3(x - 2) = (x - 2)(5x^2 - 3)$

   b. The binomials $5 - y$ and $y - 5$ are opposites. Factor -1 from $5 - y$ to obtain a common binomial factor.

$$7y(5 - y) + 3(y - 5) = -7y(y - 5) + 3(y - 5) \qquad \text{Factor } -1 \text{ from } (5 - y).$$

$$= (y - 5)(-7y + 3) \qquad \text{Distributive property}$$

EXAMPLE 2 | **Factor by grouping**

Factor the polynomial.

   a. $m^3 + 7m^2 - 2m - 14$ **b.** $n^3 + 30 + 6n^2 + 5n$

Solution

   a. $m^3 + 7m^2 - 2m - 14 = (m^3 + 7m^2) + (-2m - 14)$ Group terms.

$$= m^2(m + 7) - 2(m + 7) \qquad \text{Factor each group.}$$

$$= (m + 7)(m^2 - 2) \qquad \text{Distributive property}$$

   b. $n^3 + 30 + 6n^2 + 5n = n^3 + 6n^2 + 5n + 30$ Rearrange terms.

$$= (n^3 + 6n^2) + (5n + 30) \qquad \text{Group terms.}$$

$$= n^2(n + 6) + 5(n + 6) \qquad \text{Factor each group.}$$

$$= (n + 6)(n^2 + 5) \qquad \text{Distributive property}$$

LESSON 8.8 **Study Guide** *continued*

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Exercises for Examples 1 and 2

Factor the expression.

1. $11x(x - 8) + 3(x - 8)$

2. $9x^3 + 9x^2 - 7x - 7$

3. $10x^3 + 21y - 35x^2 - 6xy$

EXAMPLE 4 ## Solve a polynomial equation

Solve the equation $7x^3 + 14x^2 = 105x$.

Solution

$7x^3 + 14x^2 = 105x$	Write original equation.
$7x^3 + 14x^2 - 105x = 0$	Subtract $105x$ from each side.
$7x(x^2 + 2x - 15) = 0$	Factor out $7x$.
$7x(x + 5)(x - 3) = 0$	Factor the trinomial.
$7x = 0$ *or* $x + 5 = 0$ *or* $x - 3 = 0$	Zero-product property
$x = 0$ *or* $x = -5$ *or* $x = 3$	Solve for x.

The roots of the equation are 0, -5, and 3.

Exercises for Example 3

Solve the equation.

4. $2c^3 + 8c^2 - 42c = 0$

5. $4x^3 + 48x^2 + 144x = 0$

6. $5r^3 + 15r = 20r^2$