LAB 8

# Simulations and the Law of Large Numbers

Consider the following problem.

Your school assigns every student a locker at random. Each locker has a three-digit number where each digit may be 0 through 9. What is the probability that you are assigned a locker with at least two matching digits, such as 272 or 844?

You can develop and conduct a simulation to model this situation and estimate the probability.

**Conducting a Simulation**

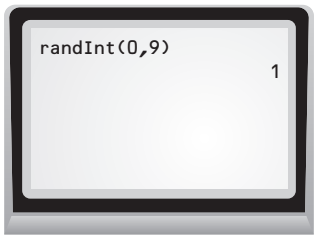
Use these steps to conduct a simulation.

1. Determine how to model the situation using number cubes, coins, a calculator, a spreadsheet, or other tools.
2. Carry out several trials of the simulation.
3. Combine your results with those of other students or carry out additional trials. Then find the experimental probability.
4. Calculate the theoretical probability, if possible, and compare it to the experimental probability.

Follow these steps to create a simulation for the problem presented above.

## STEP 1 Model the situation.

**EXAMPLE** Use the random-number feature of your calculator. Press MATH, then go to the **PRB** menu. Select **5:randInt(** . Enter **0,9)** and press ENTER to generate a random integer between 0 and 9. Repeatedly pressing ENTER generates additional random integers.




**a.** How can you use this simulation to generate one locker number?

**b.** What is the first locker number you generated?

## STEP 2 Carry out several trials of the simulation.

**EXAMPLE** The table shows one way to organize the data from the simulation. The locker numbers that are circled are those with at least two matching digits.

| Trial number | Locker number |
|---|---|
| 1 | 167 |
| 2 | 985 |
| 3 | (343) |
| 4 | 208 |

**a.** Do 10 trials of the simulation and record your data.

**b.** How many of the locker numbers have at least two matching digits?

**c.** Based on your data, what is the experimental probability that a locker number has at least two matching digits?

**STEP 3** **The Law of Large Numbers states that as the number of trials of a simulation increases, the experimental probability tends to get closer to the theoretical probability. For this reason, you should combine your data with those of other students to get a more accurate estimate of the experimental probability.**

**EXAMPLE** If there are 200 trials in the combined data and 58 of the locker numbers have at least two matching digits, then the experimental probability that a locker number has two matching digits is $\frac{58}{200} = 0.29$, or 29%.

**a.** What is the total number of trials in your combined data?

**b.** How many of the locker numbers in your combined data have at least two matching digits?

**c.** Based on your combined data, what is the experimental probability that a locker number has at least two matching digits?

**STEP 4** **Calculate the theoretical probability that a locker number has at least two matching digits.**

**EXAMPLE** The sample space consists of all possible locker numbers. There are 10 possibilities for the first digit (0 through 9), 10 possibilities for the second digit, and 10 possibilities for the third digit. By the counting principle, there are $10 \times 10 \times 10 = 1000$ outcomes in the sample space.

**a.** To determine the number of favorable outcomes, first determine how many locker numbers consist of three matching numbers.

**b.** Now determine how many locker numbers consist of exactly two matching numbers. (*Hint*: Determine how many locker numbers have exactly two 0s. Then use the same reasoning to find out how many locker numbers have exactly two 1s, exactly two 2s, and so on.)

| Locker numbers with exactly two 0s |
|---|
| 00_ |
| 0_0 |
| _00 |

**c.** What is the theoretical probability that a locker number has at least two matching digits? How does this compare to the experimental probability you found in Step 3c?

## What Do You Think?

**1.** Describe what would happen to the experimental probability you found in Step 3(c) if you conducted 5000 trials of the simulation.

**2.** A student conducts the above simulation and finds that 12 locker numbers in a row do not contain matching digits. The student claims that a locker number with matching digits is overdue and that there is a very high probability that the next number generated will have matching digits. Do you agree? Explain.

**3.** A bag contains tiles with the letters E, N, and T. You choose one tile at a time without looking. Describe and conduct a simulation to estimate the probability of choosing the letters in an order that spells an English word.