

## REVIEW KEY VOCABULARY

- quadratic function
- standard form of a quadratic function
- parabola
- parent quadratic function
- vertex of a parabola
- axis of symmetry
- minimum value
- maximum value
- intercept form of a quadratic function
- quadratic equation
- standard form of a quadratic equation
- completing the square
- vertex form of a quadratic function
- quadratic formula

## VOCABULARY EXERCISES

1. Copy and complete: The line that passes through the vertex and divides a parabola into two symmetric parts is called the   ?  .

Tell whether the function has a *minimum value* or a *maximum value*.

2.  $f(x) = 5x^2 - 4x$

3.  $f(x) = -x^2 + 6x + 2$

4.  $f(x) = 0.3x^2 - 7.7x + 1.8$

## REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

## 9.1

Graph  $y = ax^2 + c$ 

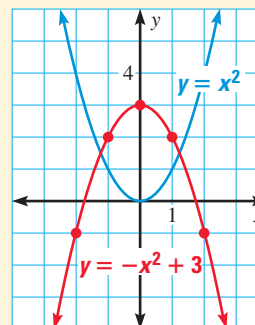
## EXAMPLE

Graph  $y = -x^2 + 3$ . Compare the graph with the graph of  $y = x^2$ .

Make a table of values for  $y = -x^2 + 3$ . Then plot the points from the table and draw a smooth curve through the points.

x	-2	-1	0	1	2
y	-1	2	3	2	-1

Both graphs have the same axis of symmetry,  $x = 0$ . However, the graph of  $y = -x^2 + 3$  has a different vertex than the graph of  $y = x^2$ , and it opens down. This is because the graph of  $y = -x^2 + 3$  is a vertical translation (of 3 units up) and a reflection in the  $x$ -axis of the graph of  $y = x^2$ .



## EXAMPLES

## 1, 2, and 4

for Exs. 5–7

## EXERCISES

Graph the function. Compare the graph with the graph of  $y = x^2$ .

5.  $y = -4x^2$

6.  $y = \frac{1}{3}x^2$

7.  $y = 2x^2 - 1$

## 9.2 Graph $y = ax^2 + bx + c$

### EXAMPLE

Graph  $y = -x^2 + 2x + 1$ .

**STEP 1 Determine** whether the parabola opens up or down. Because  $a < 0$ , the parabola opens down.

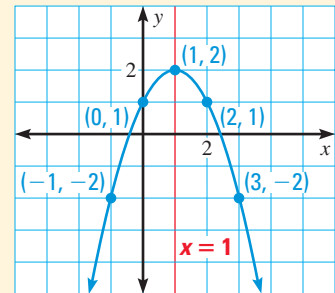
**STEP 2 Find** and draw the axis of symmetry:

$$x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$$

**STEP 3 Find** and plot the vertex. The  $x$ -coordinate of the vertex is  $-\frac{b}{2a}$ , or 1. The  $y$ -coordinate of the vertex is  $y = -(1)^2 + 2(1) + 1 = 2$ .

**STEP 4 Plot** four more points. Evaluating the function for  $x = 0$  and  $x = -1$  gives the points  $(0, 1)$  and  $(-1, -2)$ . Plot these points and their reflections in the axis of symmetry.

**STEP 5 Draw** a parabola through the plotted points.



**EXAMPLE 2**  
for Exs. 8–10

### EXERCISES

Graph the function. Label the vertex and axis of symmetry.

8.  $y = x^2 + 4x + 1$

9.  $y = 2x^2 - 4x - 3$

10.  $y = -2x^2 + 8x + 5$

## 9.3 Solve Quadratic Equations by Graphing

### EXAMPLE

Solve  $x^2 - 7x = -12$  by graphing.

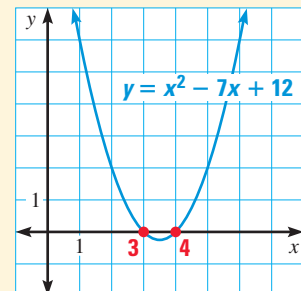
**STEP 1 Write** the equation in standard form.

$$x^2 - 7x = -12 \quad \text{Write original equation.}$$

$$x^2 - 7x + 12 = 0 \quad \text{Add 12 to each side.}$$

**STEP 2 Graph** the related function  $y = x^2 - 7x + 12$ . The  $x$ -intercepts of the graph are 3 and 4.

► The solutions of the equation  $x^2 - 7x + 12 = 0$  are 3 and 4.



**EXAMPLES**  
**1, 2, and 3**  
for Exs. 11–13

### EXERCISES

Solve the equation by graphing.

11.  $4x^2 + x + 3 = 0$

12.  $x^2 + 2x = -1$

13.  $-x^2 + 8 = 7x$

## 9.4 Use Square Roots to Solve Quadratic Equations

**EXAMPLE**Solve  $5(x - 6)^2 = 30$ . Round the solutions to the nearest hundredth.

$$5(x - 6)^2 = 30 \quad \text{Write original equation.}$$

$$(x - 6)^2 = 6 \quad \text{Divide each side by 5.}$$

$$x - 6 = \pm \sqrt{6} \quad \text{Take square roots of each side.}$$

$$x = 6 \pm \sqrt{6} \quad \text{Add 6 to each side.}$$

► The solutions of the equation are  $6 + \sqrt{6} \approx 8.45$  and  $6 - \sqrt{6} \approx 3.55$ .

**EXERCISES**

Solve the equation. Round your solutions to the nearest hundredth, if necessary.

14.  $6x^2 - 54 = 0$

15.  $3x^2 + 7 = 4$

16.  $g^2 + 11 = 24$

17.  $7n^2 + 5 = 9$

18.  $2(a + 7)^2 = 34$

19.  $3(w - 4)^2 = 5$

**EXAMPLES**

1–4

for Exs. 14–19

## 9.5 Solve Quadratic Equations by Completing the Square

**EXAMPLE**Solve  $3x^2 + 12x = 18$  by completing the square.

$$3x^2 + 12x = 18 \quad \text{Write original equation.}$$

$$x^2 + 4x = 6 \quad \text{Divide each side by 3.}$$

$$x^2 + 4x + 2^2 = 6 + 2^2 \quad \text{Add } \left(\frac{4}{2}\right)^2, \text{ or } 2^2, \text{ to each side.}$$

$$(x + 2)^2 = 10 \quad \text{Write left side as the square of a binomial.}$$

$$x + 2 = \pm \sqrt{10} \quad \text{Take square roots of each side.}$$

$$x = -2 \pm \sqrt{10} \quad \text{Subtract 2 from each side.}$$

► The solutions of the equation are  $-2 + \sqrt{10} \approx 1.16$  and  $-2 - \sqrt{10} \approx -5.16$ .

**EXERCISES**

Solve the equation by completing the square. Round your solutions to the nearest hundredth, if necessary.

20.  $x^2 - 14x = 51$

21.  $2a^2 + 12a - 4 = 0$

22.  $2n^2 + 4n + 1 = 10n + 9$

23.  $5g^2 - 3g + 6 = 2g^2 + 9$

**EXAMPLES**

2 and 3

for Exs. 20–23

## 9.6 Solve Quadratic Equations by the Quadratic Formula

### EXAMPLE

Solve  $4x^2 + 3x = 1$ .

$$4x^2 + 3x = 1$$

$$4x^2 + 3x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-3 \pm \sqrt{25}}{8} = \frac{-3 \pm 5}{8}$$

Write original equation.

Write in standard form.

Quadratic formula

Substitute values in the quadratic formula:

$a = 4$ ,  $b = 3$ , and  $c = -1$ .

Simplify.

► The solutions of the equation are  $\frac{-3 + 5}{8} = \frac{1}{4}$  and  $\frac{-3 - 5}{8} = -1$ .

### EXERCISES

Use the quadratic formula to solve the equation. Round your solutions to the nearest hundredth, if necessary.

24.  $x^2 - 2x - 15 = 0$

25.  $2m^2 + 7m - 3 = 0$

26.  $-w^2 + 5w = 3$

27.  $5n^2 - 7n = -1$

28.  $t^2 - 4 = 6t + 8$

29.  $2h - 1 = 10 - 9h^2$

### EXAMPLES

1-3

for Exs. 24-29

## 9.7 Solve Systems with Quadratic Equations

### EXAMPLE

Solve the system using substitution:  $y = x + 10$

Equation 1

$$y = x^2 + x + 1$$

Equation 2

**STEP 1** Solve one of the equations for  $y$ . Equation 1 is already solved for  $y$ .

**STEP 2** Substitute  $x + 10$  for  $y$  in Equation 2 and solve for  $x$ .

$$y = x^2 + x + 1$$

Write original equation 2.

$$x + 10 = x^2 + x + 1$$

Substitute  $x + 10$  for  $y$ .

$$9 = x^2$$

Solve for  $x^2$

$$x = -3 \text{ or } x = 3$$

Solve for  $x$

**STEP 3** Substitute both  $-3$  and  $3$  for  $y$  in Equation 1

$$y = -3 + 10 = 7 \text{ and } y = 3 + 10 = 13$$

► The solutions of the system are  $(-3, 7)$  and  $(3, 13)$ .

### EXERCISES

Solve the system using the substitution method or a graphing calculator.

30.  $y = x + 8$

31.  $x + y = 0$

32.  $2x + y = 1$

$$y = x^2 + 2x + 2$$

$$y = 2x^2 - 3x - 4$$

$$y = 3x^2 - x - 1$$

33. Solve  $4x^2 - 2x - 1 = 2x + 2$  using a system of equations.

### EXAMPLES

1-3

for Exs. 30-33

## 9.8 Compare Linear, Exponential, and Quadratic Models

## EXAMPLE

Use differences or ratios to tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*.

a.

<b>x</b>	-1	0	1	2
<b>y</b>	5	3	1	-1

Differences:  $-2$   $-2$   $-2$

► The table of values represents a linear function.

b.

<b>x</b>	-1	0	1	2
<b>y</b>	4	5	4	1

First differences:  $1$   $-1$   $-3$

Second differences:  $-2$   $-2$

► The table of values represents a quadratic function.

## EXERCISES

Tell whether the table of values represents a *linear function*, an *exponential function*, or a *quadratic function*.

34.

<b>x</b>	1	2	3	4	5	6
<b>y</b>	1	2	4	8	16	32

35.

<b>x</b>	-2	-1	0	1	2	3
<b>y</b>	0	3	6	9	12	15

## 9.9 Model Relationships

## EXAMPLE

Decide which function is increasing more rapidly.

Linear Function 1 has an  $x$ -intercept of  $-3$  and a  $y$ -intercept of  $2$ .

Linear Function 2 includes the points in the table below.

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-3	-2.5	-2	-1.5	-1

The points  $(-3, 0)$  and  $(0, 2)$  are on the graph of Linear Function 1, so its slope is  $\frac{2-0}{0-(-3)} = \frac{2}{3}$ . The table for Linear Function 2 shows that for each increase of 1 in the value of  $x$ , there is an increase of 0.5 in the value of  $y$ . The slope of the graph of Linear Function 2 is  $= \frac{0.5}{1} = \frac{1}{2}$ . So, Linear Function 1 is increasing more rapidly.

## EXERCISES

36. Linear Function 1 has a  $y$ -intercept of 3 and a slope of  $-1$ . Linear Function 2 has an  $x$ -intercept of 4 and a  $y$ -intercept of 3. Which linear function is decreasing more rapidly?
37. The population of a city is increasing at a rate of 2.5% per decade. What type of function would be a good model for this situation?

EXAMPLE 2  
for Exs. 34–35

EXAMPLES  
1–2  
for Exs. 36–37