

CHAPTER
9

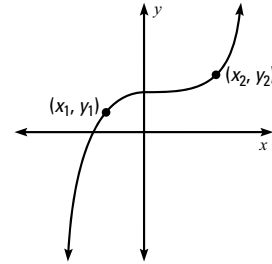
Average Rates of Change of Quadratic Functions

Geometrically, the **average rate of change** of a function is equal to the slope of the line through two specified points on the graph of the function.

KEY CONCEPT
Average Rate of Change

Suppose (x_1, y_1) and (x_2, y_2) are two points on a graph of a function, and that $x_1 < x_2$.

The average rate of change from x_1 to x_2 is equal to $\frac{y_2 - y_1}{x_2 - x_1}$.


EXAMPLE 1 Find the rate of change of a quadratic function

Given the quadratic function with equation $y = x^2 + 4$, find the average rate of change from $x = 1$ to $x = 3$.

Solution:

When $x = 1$, $y = 5$ and when $x = 3$, $y = 13$.

Average rate of change from $x = 1$ to $x = 3$ is equal to $\frac{13 - 5}{3 - 1} = \frac{8}{2} = 4$. ■

EXAMPLE 2 Compare slope of a line and average rate of change

The slope of the line with equation $y = 2x - 1$ is 2. Find the average rate of change from $x = a$ to $x = b$. Show that the average rate of change is equal to the slope of the line.

Solution:

The average rate of change is $\frac{(2b - 1) - (2a - 1)}{b - a} = \frac{2b - 2a}{b - a} = \frac{2(b - a)}{(b - a)} = 2$

The slope of the line is equal to the average rate of change. ■

EXAMPLE 3 Find the interval given the average rate of change

Given the quadratic function with equation $y = x^2 + 2x$, find the value a for which the average rate of change from $x = 0$ to $x = a$ is equal to 4.

Solution:

When $x = 0$, $y = 0$ and when $x = a$, $y = a^2 + 2a$.

Average rate of change from $x = 0$ to $x = a$ is equal to $\frac{a^2 + 2a - 0}{a - 0} = \frac{a^2 + 2a}{a} = a + 2$.
Therefore, $a + 2 = 4$ or $a = 2$. ■

Average Rates of Change of Quadratic Functions *continued*

Practice

Find the average rate of change from $x = -1$ to $x = 2$ for the function.

1. $y = x^2$ 2. $y = 3^x$ 3. $y = -\frac{1}{2}x - 4$ 4. $y = x^2 - 2x + 3$
 5. $y = \left(\frac{1}{2}\right)^x - 1$ 6. $y = 2x^2 - 1$ 7. $y = -10x$ 8. $y = (5 - x)^2$

Find the average rate of change of $y = 2x^2 + x$ over the specified interval.

9. $x = 2$ to $x = 4$ 10. $x = -2$ to $x = -4$
 11. $x = 0.25$ to $x = 1.25$ 12. $x = -\frac{1}{2}$ to $x = 0$

If $a > 0$, find the value of a for which the average rate of change from $x = 0$ to $x = a$ is equal to 2.

13. $y = 4x^2$ 14. $y = \frac{1}{2}x^2$
 15. $y = x^2 + x + \frac{1}{2}$ 16. $y = (x - 6)^2$

If $a < 0$, find the value of a for which the average rate of change from $x = a$ to $x = 0$ is equal to $-\frac{1}{2}$.

17. $y = 2x^2$ 18. $y = x^2 + x$ 19. $y = (x + 1)^2$ 20. $y = x^2 - 1$

Find the average rate of change of the graph of the function from $x = -3$ to $x = -1$.

