9.3 Solve Quadratic Equations by Graphing

Before	You solved quadratic equations by factoring.
Now	You will solve quadratic equations by graphing.
Why?	So you can solve a problem about sports, as in Example 6.

Key Vocabulary
- **quadratic equation**
- ***x*-intercept**
- **roots**
- **zero of a function**

A **quadratic equation** is an equation that can be written in the **standard form** $ax^2 + bx + c = 0$ where $a \neq 0$.

You have used factoring to solve a quadratic equation. You can also use graphing to solve a quadratic equation. Notice that the solutions of the equation $ax^2 + bx + c = 0$ are the x-intercepts of the graph of the related function $y = ax^2 + bx + c$.

COMMON CORE

CC.9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.*

Solve by Factoring

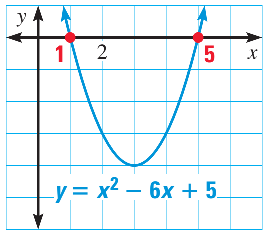
$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

$$x = 1 \quad or \quad x = 5$$

Solve by Graphing

To solve $x^2 - 6x + 5 = 0$, graph $y = x^2 - 6x + 5$. From the graph you can see that the x-intercepts are 1 and 5.



READING
In this course, *solutions* refers to real-number solutions.

To solve a quadratic equation by graphing, first write the equation in standard form, $ax^2 + bx + c = 0$. Then graph the related function $y = ax^2 + bx + c$. The x-intercepts of the graph are the solutions, or roots, of $ax^2 + bx + c = 0$.

EXAMPLE 1 Solve a quadratic equation having two solutions

Solve $x^2 - 2x = 3$ by graphing.

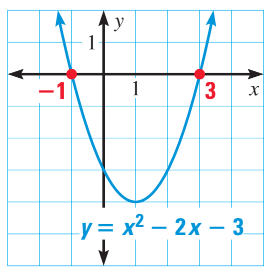
Solution

STEP 1 **Write** the equation in standard form.

$x^2 - 2x = 3$ **Write original equation.**

$x^2 - 2x - 3 = 0$ **Subtract 3 from each side.**

STEP 2 **Graph** the function $y = x^2 - 2x - 3$. The x-intercepts are -1 and 3.



▶ The solutions of the equation $x^2 - 2x = 3$ are -1 and 3.

CHECK You can check -1 and 3 in the original equation.

$x^2 - 2x = 3$	$x^2 - 2x = 3$	**Write original equation.**
$(-1)^2 - 2(-1) \stackrel{?}{=} 3$	$(3)^2 - 2(3) \stackrel{?}{=} 3$	**Substitute for x.**
$3 = 3$ ✓	$3 = 3$ ✓	**Simplify. Each solution checks.**

EXAMPLE 2 Solve a quadratic equation having one solution

Solve $-x^2 + 2x = 1$ by graphing.

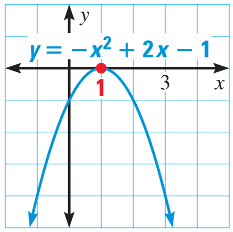
Solution

STEP 1 **Write** the equation in standard form.

$-x^2 + 2x = 1$ — **Write original equation.**

$-x^2 + 2x - 1 = 0$ — **Subtract 1 from each side.**

STEP 2 **Graph** the function $y = -x^2 + 2x - 1$. The x-intercept is 1.



▶ The solution of the equation $-x^2 + 2x = 1$ is 1.

EXAMPLE 3 Solve a quadratic equation having no solution

Solve $x^2 + 7 = 4x$ by graphing.

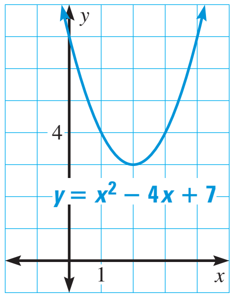
Solution

STEP 1 **Write** the equation in standard form.

$x^2 + 7 = 4x$ — **Write original equation.**

$x^2 - 4x + 7 = 0$ — **Subtract $4x$ from each side.**

STEP 2 **Graph** the function $y = x^2 - 4x + 7$. The graph has no x-intercepts.



▶ The equation $x^2 + 7 = 4x$ has no solution.

AVOID ERRORS

Do not confuse y-intercepts and x-intercepts. Although the graph has a y-intercept, it does not have any x-intercepts.

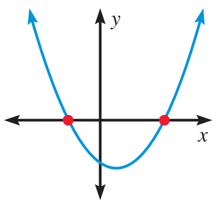
✓ GUIDED PRACTICE for Examples 1, 2, and 3

Solve the equation by graphing.

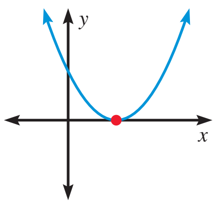
1. $x^2 - 6x + 8 = 0$ **2.** $x^2 + x = -1$ **3.** $-x^2 + 6x = 9$

KEY CONCEPT *For Your Notebook*

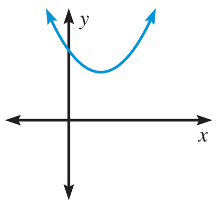
Number of Solutions of a Quadratic Equation



A quadratic equation has **two solutions** if the graph of its related function has **two x-intercepts**.



A quadratic equation has **one solution** if the graph of its related function has **one x-intercept**.



A quadratic equation has **no real solution** if the graph of its related function has **no x-intercepts**.

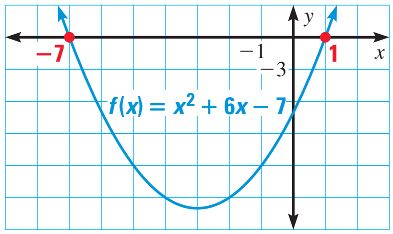
FINDING ZEROS Because a zero of a function is an x-intercept of the function's graph, you can use the function's graph to find the zeros of a function.

EXAMPLE 4 Find the zeros of a quadratic function

Find the zeros of $f(x) = x^2 + 6x - 7$.

ANOTHER WAY
You can find the zeros of a function by factoring:
$f(x) = x^2 + 6x - 7$
$0 = x^2 + 6x - 7$
$0 = (x + 7)(x - 1)$
$x = -7$ *or* $x = 1$

Solution



Graph the function $f(x) = x^2 + 6x - 7$. The x-intercepts are -7 and 1.

▶ The zeros of the function are -7 and 1.

CHECK Substitute -7 and 1 in the original function.

$f(-7) = (-7)^2 + 6(-7) - 7 = 0$ ✓

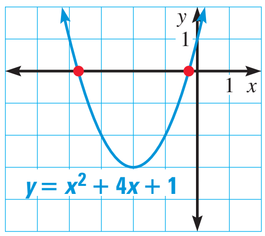
$f(1) = (1)^2 + 6(1) - 7 = 0$ ✓

APPROXIMATING ZEROS The zeros of a function are not necessarily integers. To approximate zeros, look at the signs of the function values. If two function values have opposite signs, then a zero falls between the x-values that correspond to the function values.

EXAMPLE 5 Approximate the zeros of a quadratic function

Approximate the zeros of $f(x) = x^2 + 4x + 1$ to the nearest tenth.

Solution



STEP 1 **Graph** the function $f(x) = x^2 + 4x + 1$. There are two x-intercepts: one between -4 and -3 and another between -1 and 0.

STEP 2 **Make** a table of values for x-values between -4 and -3 and between -1 and 0 using an increment of 0.1. Look for a change in the signs of the function values.

INTERPRET FUNCTION VALUES
The function value that is closest to 0 indicates the x-value that best approximates a zero of the function.

x	−3.9	−3.8	−3.7	−3.6	−3.5	−3.4	−3.3	−3.2	−3.1
$f(x)$	0.61	0.24	−0.11	−0.44	−0.75	−1.04	−1.31	−1.56	−1.79

x	−0.9	−0.8	−0.7	−0.6	−0.5	−0.4	−0.3	−0.2	−0.1
$f(x)$	−1.79	−1.56	−1.31	−1.04	−0.75	−0.44	−0.11	0.24	0.61

▶ In each table, the function value closest to 0 is -0.11. So, the zeros of $f(x) = x^2 + 4x + 1$ are about -3.7 and about -0.3.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Find the zeros of $f(x) = x^2 + x - 6$.

5. Approximate the zeros of $f(x) = -x^2 + 2x + 2$ to the nearest tenth.

EXAMPLE 6 Solve a multi-step problem

SPORTS An athlete throws a shot put with an initial vertical velocity of 40 feet per second as shown.

a. Write an equation that models the height h (in feet) of the shot put as a function of the time t (in seconds) after it is thrown.

b. Use the equation to find the time that the shot put is in the air.

Solution

a. Use the initial vertical velocity and the release height to write a vertical motion model.

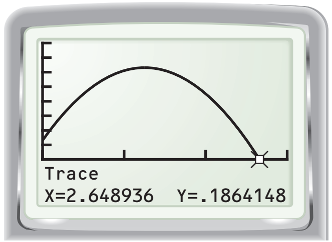
$h = -16t^2 + vt + s$ **Vertical motion model**

$h = -16t^2 + 40t + 6.5$ **Substitute 40 for *v* and 6.5 for *s*.**

b. The shot put lands when $h = 0$. To find the time t when $h = 0$, solve $0 = -16t^2 + 40t + 6.5$ for t.

To solve the equation, graph the related function $h = -16t^2 + 40t + 6.5$ on a graphing calculator. Use the *trace* feature to find the t-intercepts.

▸ There is only one positive t-intercept. The shot put is in the air for about 2.6 seconds.



USE A GRAPHING CALCULATOR
When entering $h = -16t^2 + 40t + 6.5$ in a graphing calculator, use y instead of h and x instead of t.

✓ GUIDED PRACTICE for Example 6

6. **WHAT IF?** In Example 6, suppose the initial vertical velocity is 30 feet per second. Find the time that the shot put is in the air.

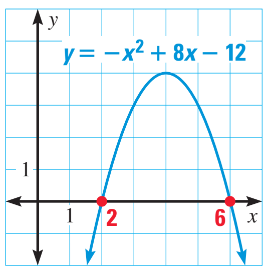
CONCEPT SUMMARY *For Your Notebook*

Relating Solutions of Equations, *x*-Intercepts of Graphs, and Zeros of Functions

Solutions of an Equation
The solutions of the equation $-x^2 + 8x - 12 = 0$ are 2 and 6.

***x*-Intercepts of a Graph**
The x-intercepts of the graph of $y = -x^2 + 8x - 12$ occur where $y = 0$, so the x-intercepts are 2 and 6, as shown.

Zeros of a Function
The zeros of the function $f(x) = -x^2 + 8x - 12$ are the values of x for which $f(x) = 0$, so the zeros are 2 and 6.



9.3 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 5 and 51

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 46, 53, and 54

SKILL PRACTICE

1. **VOCABULARY** Write $2x^2 + 11 = 9x$ in standard form.

2. ★ **WRITING** Is $3x^2 - 2 = 0$ a quadratic equation? *Explain.*

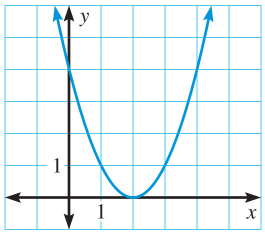
EXAMPLES 1, 2, and 3 for Exs. 3–21

SOLVING EQUATIONS **Solve the equation by graphing.**

3. $x^2 - 5x + 4 = 0$
4. $x^2 + 5x + 6 = 0$
5. $x^2 + 6x = -8$
6. $x^2 - 4x = 5$
7. $x^2 - 16 = 6x$
8. $x^2 - 12x = -35$
9. $x^2 - 6x + 9 = 0$
10. $x^2 + 8x + 16 = 0$
11. $x^2 + 10x = -25$
12. $x^2 + 81 = 18x$
13. $-x^2 - 14x = 49$
14. $-x^2 + 16x = 64$
15. $x^2 - 5x + 7 = 0$
16. $x^2 - 2x + 3 = 0$
17. $x^2 + x = -2$
18. $\frac{1}{5}x^2 - 5 = 0$
19. $\frac{1}{2}x^2 + 2x = 6$
20. $-\frac{1}{4}x^2 - 8 = x$

21. **ERROR ANALYSIS** The graph of the function related to the equation $0 = x^2 - 4x + 4$ is shown. *Describe* and correct the error in solving the equation.

> The only solution of the equation $0 = x^2 - 4x + 4$ is 4.



EXAMPLE 4 for Exs. 22–30

FINDING ZEROS **Find the zeros of the function.**

22. $f(x) = x^2 + 4x - 5$
23. $f(x) = x^2 - x - 12$
24. $f(x) = x^2 - 5x - 6$
25. $f(x) = x^2 + 3x - 10$
26. $f(x) = -x^2 + 8x + 9$
27. $f(x) = x^2 + x - 20$
28. $f(x) = -x^2 - 7x + 8$
29. $f(x) = x^2 - 12x + 11$
30. $f(x) = -x^2 + 4x + 12$

SOLVING EQUATIONS **Solve the equation by graphing.**

31. $2x^2 + x = 3$
32. $4x^2 - 5 = 8x$
33. $4x^2 - 4x + 1 = 0$
34. $x^2 + x = -\frac{1}{4}$
35. $3x^2 + 1 = 2x$
36. $5x^2 + x + 3 = 0$

EXAMPLE 5 for Exs. 37–46

APPROXIMATING ZEROS **Approximate the zeros of the function to the nearest tenth.**

37. $f(x) = x^2 + 4x + 2$
38. $f(x) = x^2 - 5x + 3$
39. $f(x) = x^2 - 2x - 5$
40. $f(x) = -x^2 - 3x + 3$
41. $f(x) = -x^2 + 7x - 5$
42. $f(x) = -x^2 - 5x - 2$
43. $f(x) = 2x^2 + x - 2$
44. $f(x) = -3x^2 + 8x - 2$
45. $f(x) = 5x^2 + 30x + 30$

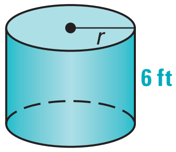
46. ★ **MULTIPLE CHOICE** Which function has a zero between −3 and −2?

(A) $f(x) = -3x^2 + 4x + 11$ (B) $f(x) = 4x^2 - 3x - 11$

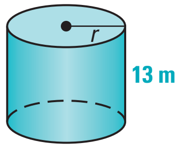
(C) $f(x) = 3x^2 + 4x - 11$ (D) $f(x) = 3x^2 + 11$

CHALLENGE **Use the given surface area S of the cylinder to find the radius r to the nearest tenth. (Use 3.14 for π.)**

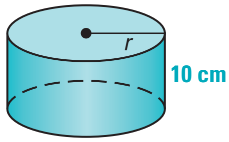
47. $S = 251 \text{ ft}^2$



48. $S = 716 \text{ m}^2$



49. $S = 1074 \text{ cm}^2$



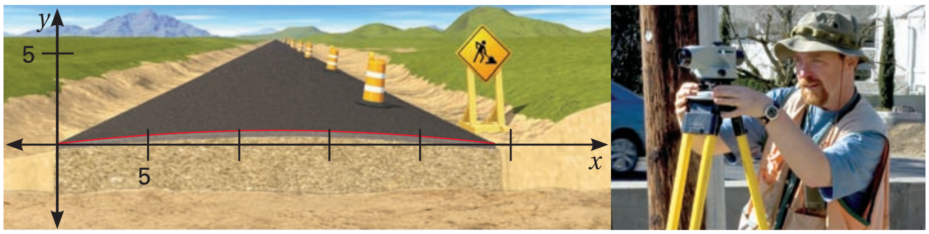
PROBLEM SOLVING

GRAPHING CALCULATOR **You may wish to use a graphing calculator to complete the following Problem Solving exercises.**

EXAMPLE 6 for Exs. 50–52

50. SOCCER The height y (in feet) of a soccer ball after it is kicked can be modeled by the graph of the equation $y = -0.04x^2 + 1.2x$ where x is the horizontal distance (in feet) that the ball travels. The ball is not touched, and it lands on the ground. Find the distance that the ball was kicked.

51. SURVEYING To keep water off a road, the road's surface is shaped like a parabola as in the cross section below. The surface of the road can be modeled by the graph of $y = -0.0017x^2 + 0.041x$ where x and y are measured in feet. Find the width of the road to the nearest tenth of a foot.



52. DIVING During a cliff diving competition, a diver begins a dive with his center of gravity 70 feet above the water. The initial vertical velocity of his dive is 8 feet per second.

a. Write an equation that models the height h (in feet) of the diver's center of gravity as a function of time t (in seconds).

b. How long after the diver begins his dive does his center of gravity reach the water?

53. ★ **SHORT RESPONSE** An arc of water sprayed from the nozzle of a fountain can be modeled by the graph of $y = -0.75x^2 + 6x$ where x is the horizontal distance (in feet) from the nozzle and y is the vertical distance (in feet). The diameter of the circle formed by the arcs on the surface of the water is called the display diameter. Find the display diameter of the fountain. *Explain* your reasoning.

54. ★ **EXTENDED RESPONSE** Two softball players are practicing catching fly balls. One player throws a ball to the other. She throws the ball upward from a height of 5.5 feet with an initial vertical velocity of 40 feet per second for her teammate to catch.

a. Write an equation that models the height h (in feet) of the ball as a function of time t (in seconds) after it is thrown.

b. If her teammate misses the ball and it lands on the ground, how long was the ball in the air?

c. If her teammate catches the ball at a height of 5.5 feet, how long was the ball in the air? *Explain* your reasoning.

55. **CHALLENGE** A stream of water from a fire hose can be modeled by the graph of $y = -0.003x^2 + 0.58x + 3$ where x and y are measured in feet. A firefighter is holding the hose 3 feet above the ground, 137 feet from a building. Will the stream of water pass through a window if the top of the window is 26 feet above the ground? *Explain.*

QUIZ

Graph the function. Compare the graph with the graph of $y = x^2$.

1. $y = -\frac{1}{2}x^2$ **2.** $y = 2x^2 - 5$ **3.** $y = -x^2 + 3$

Graph the function. Label the vertex and axis of symmetry.

4. $y = x^2 + 5$ **5.** $y = -5x^2 + 1$

6. $y = x^2 + 4x - 2$ **7.** $y = 2x^2 - 12x + 5$

8. $y = -\frac{1}{2}x^2 + 2x - 5$ **9.** $y = -4x^2 - 10x + 2$

Solve the equation by graphing.

10. $x^2 - 7x = 8$ **11.** $x^2 + 6x + 9 = 0$ **12.** $x^2 + 10x = 11$

13. $x^2 - 7 = -6x$ **14.** $-x^2 + x - 1 = 0$ **15.** $x^2 - 4x + 9 = 0$

Find the zeros of the function.

16. $f(x) = x^2 + 3x - 10$ **17.** $f(x) = x^2 - 8x + 12$ **18.** $f(x) = -x^2 + 5x + 14$

See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com